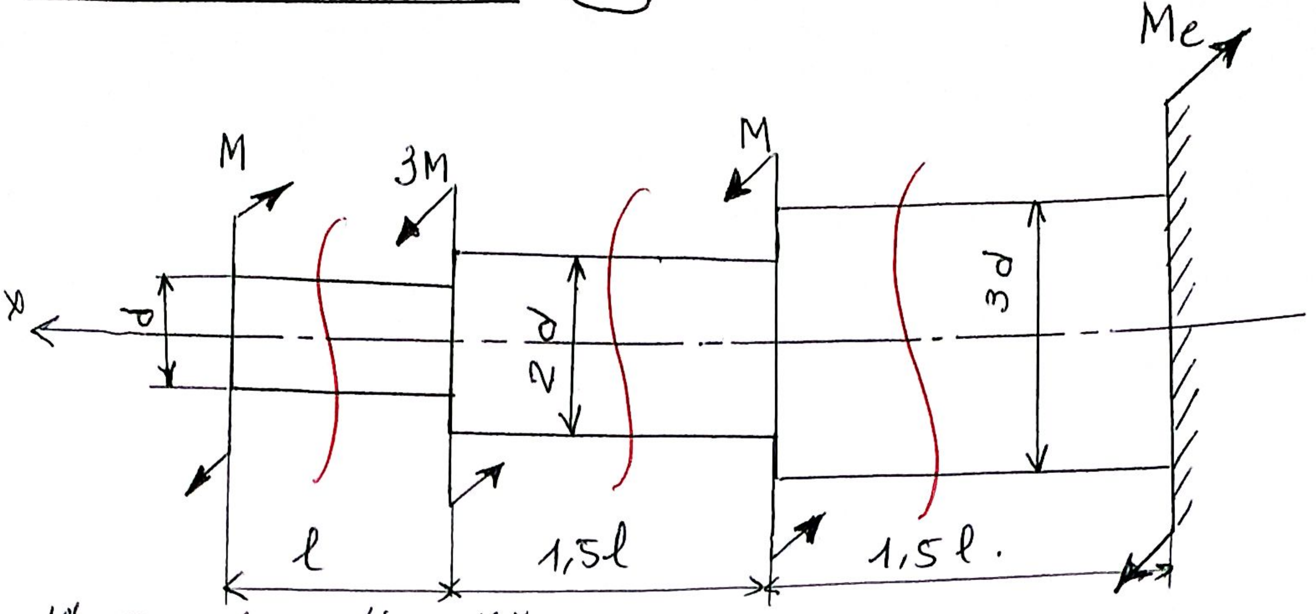
Solution de l'exercice (2b)

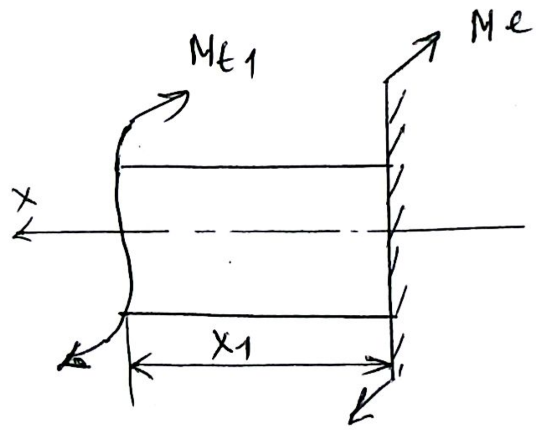


1) Equation d'équilibre

$$\sum M/x = 0 \Rightarrow -Me + M + 3M - M = 0.$$

$$Me = 3M.$$

Tronçon I $\quad 0 \leq X_1 \leq 1{,}5l.$



$$\sum M/x = 0 \Rightarrow -M_{t_1} - Me = 0.$$

$$M_{t1} = -Me = -3M.$$

$$\alpha_1 = \int_0^{1{,}5l} \frac{M_{t_1} dx_1}{G I_{o3}} = \frac{-3M X_1}{G I_{o3}} \Bigg|_0^{1{,}5l}.$$

Remarque:
on sait que l'angle $\alpha = 0$ à l'encastrement, c'est pour ça on commence à droite c'est-à-dire à l'encastrement.
Nous avons aussi le moment d'Inertie I_o n'est pas constant tout le long de la barre.

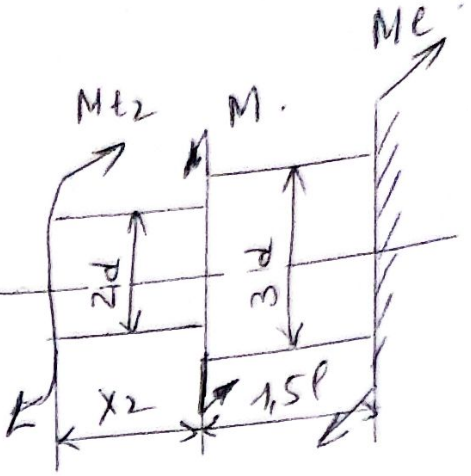
$$I_o = \frac{\pi d^4}{32}. \qquad I_{o2} = \frac{\pi (2d)^4}{32} = \frac{16\pi d^4}{32} = 16 I_o$$

$$I_{o1} = \frac{\pi d^4}{32} = I_o. \qquad I_{o3} = \frac{\pi (3d)^4}{32} = 81\frac{\pi d^4}{3} = 81 I_o.$$

pour $x_1 = 0 \Rightarrow \alpha_1 = 0$.

$$x_1 = 1{,}5\ell \Rightarrow \alpha_1 = -\frac{4{,}5M\ell}{G\underbrace{I_{03}}_{81 I_0}} = \frac{-4{,}5M\ell}{G 81 I_0} = \frac{-0{,}055M\ell}{GI_0}$$

Tronçon II $\quad 0 \leq x_2 \leq 1{,}5\ell$.



$\Sigma M/x = 0 \Rightarrow -Me + M - M_{t2} = 0$

$M_{t2} = -Me + M = -3M + M = -2M$

$M_{t2} = -2M$.

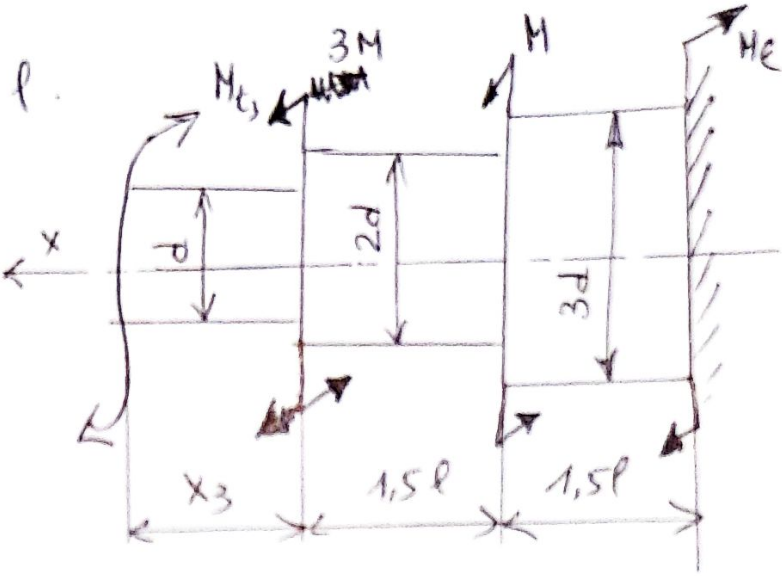
$$\alpha_2 = \int_0^{1{,}5\ell} \frac{M_{t2}\,dx_2}{GI_{02}} = \left. -\frac{2Mx}{GI_{02}} \right|_0^{1{,}5\ell}$$

$x_2 = 0 \Rightarrow \alpha_2 = 0$.

$$x_2 = 1{,}5\ell \Rightarrow \alpha_2 = -\frac{3M\ell}{G\underbrace{I_{02}}_{16 I_0}} = \frac{-3M\ell}{16 G I_0} = -\frac{0{,}1875M\ell}{GI_0}$$

$$\alpha_{1,2} = \alpha_1 + \alpha_2 = -\frac{0{,}055M\ell}{GI_0} - \frac{0{,}1875M\ell}{GI_0} = -\frac{0{,}242M\ell}{GI_0}$$

Tronçon III $\quad 0 \leq x_3 \leq \ell$.



$\Sigma M/x = 0 \Rightarrow$

$-Me + M + 3M - M_{t3} = 0 \Rightarrow$

$M_{t3} = -Me + M + 3M$

$M_{t3} = -3M + M + 3M =$

$M_{t3} = M$

$$\alpha_3 = \int_0^l \frac{M_{t3}\, dx_3}{GI_{o1}} = \frac{Mx}{GI_o}\Big|_0^l$$

pour $x_3 = 0 \Rightarrow \alpha_3 = 0$

$x_3 = l \Rightarrow \alpha_3 = \frac{Ml}{G\underset{I_o}{\overset{\parallel}{I_{o1}}}} = \frac{Ml}{GI_o}$

$$\alpha_{1,2,3} = \alpha_{1,2} + \alpha_3 = -\frac{0{,}242\,Ml}{GI_o} + \frac{Ml}{GI_o} = \frac{0{,}758\,Ml}{GI_o}.$$

La contrainte maximale tangentielle se trouve sur le tronçon I. car $M_{t1} = -3M$. on ne tient pas compte du signe (le signe indique que le sens du moment)

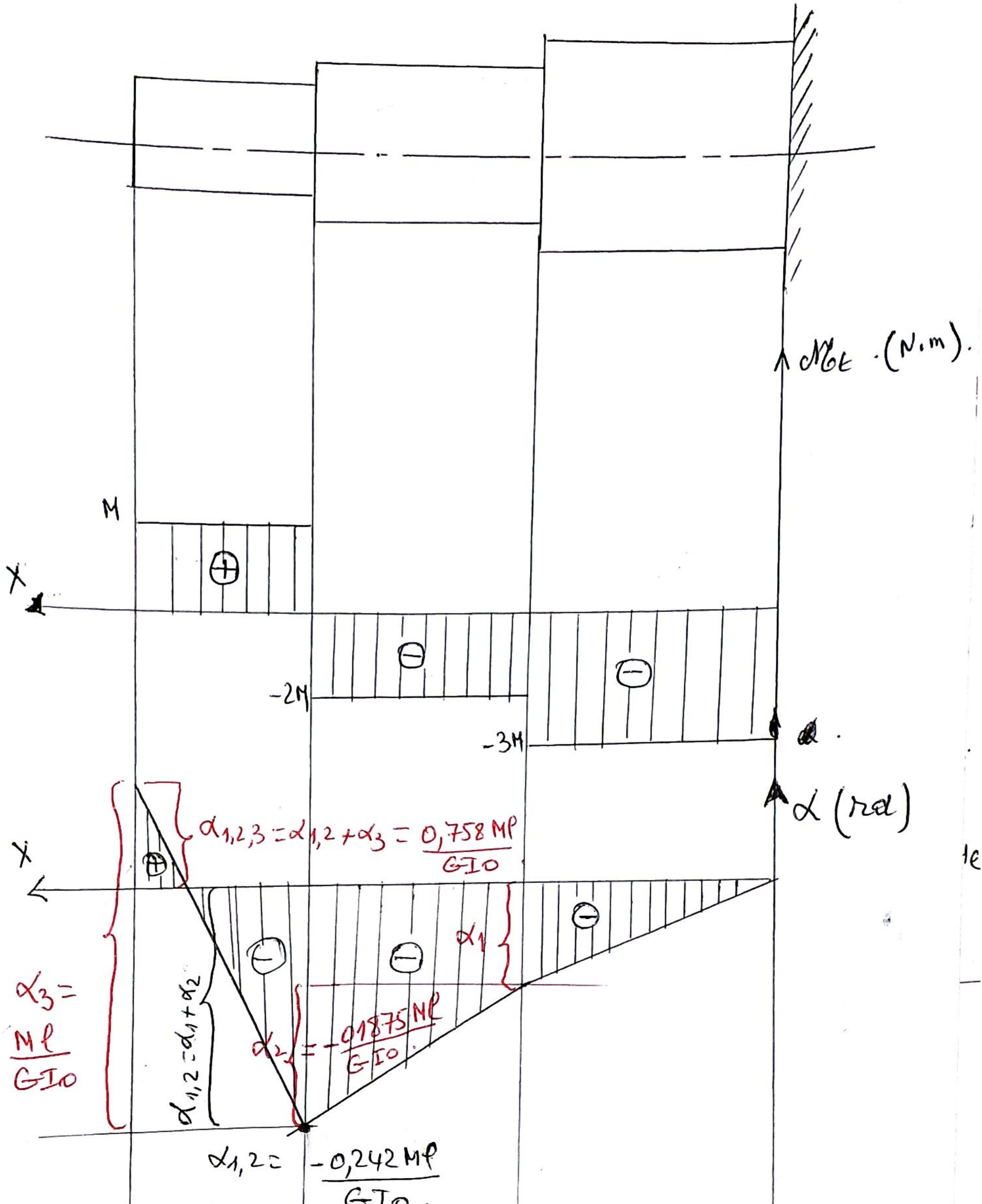
$\tau_{max} = \frac{M_{t\,max}}{W_p}$. ici $M_{t\,max} = -3M$.

W_p du tronçon sur lequel se trouve le moment de torsion maximal. ($M_{t\,max}$) ici c'est le tronçon I. donc $W_{p_1} = \frac{I_{o3}}{\frac{3d}{2}} = \frac{\frac{\pi 81 d^4}{32}}{\frac{3d}{2}} = .$

$$W_{p1} = \frac{\pi 27 d^3}{16}.$$

$$\tau_{max} = \frac{M_{t\,max}}{W_{p1}} = \frac{-3M}{\frac{\pi 27 d^3}{16}} = \frac{-48M}{27\pi d^3}.$$

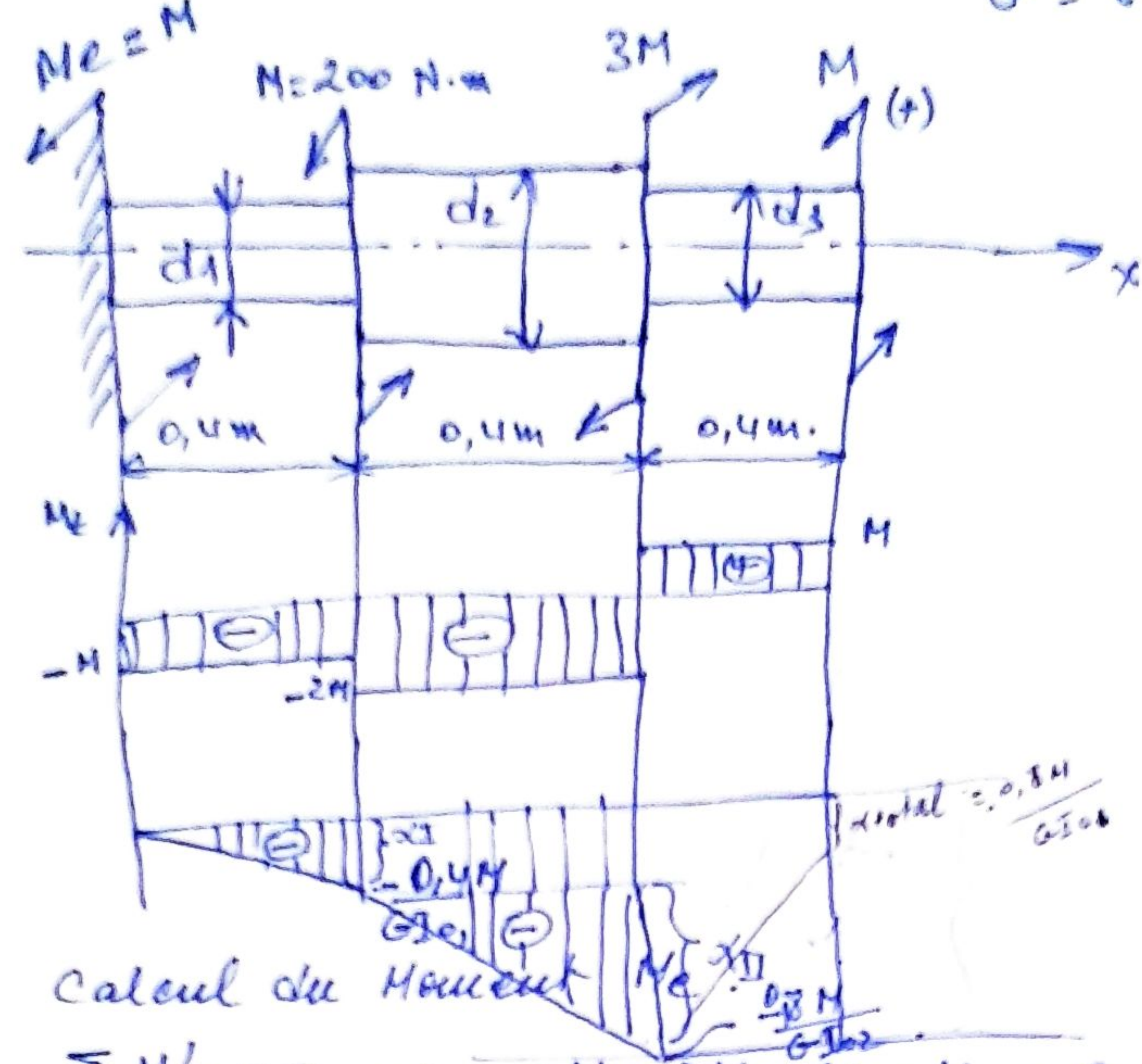
Construction des diagrammes.



Exercice N° 3 .

$[\tau] = 40\ MN/m^2$

$G = 8.10^4\ MN/m^2$

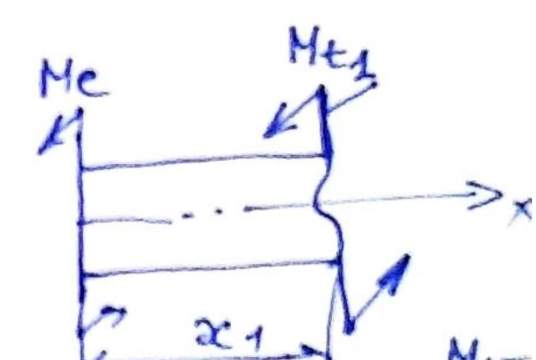


Calcul du Moment

$\Sigma M/x = 0 \Rightarrow +M - 3M + M + M_e = 0 \Rightarrow$

$M_e = 3M - 2M = M$.

Tronçon I $\quad 0 \leq X_1 \leq 0,4$



$\Sigma M/x \quad M_{t1} + M_e = 0 \Rightarrow$

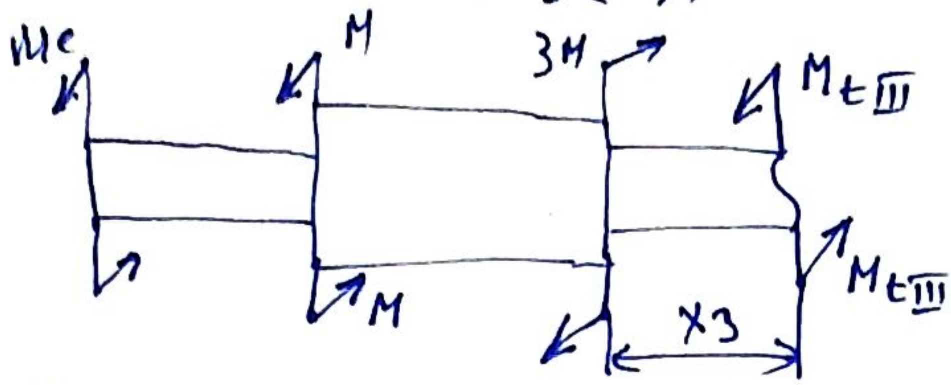
$M_{t1} = -M_e = -M$

Tronçon II $\quad 0 \leq X_2 \leq 0,4$

$\Sigma M/x = M_{tII} + M + M_e = 0$.

$M_{tII} = -2M$.

Tronçon III $0 \leq x_3 \leq 0,4$ (2)



$$\sum M/x = 0 \Rightarrow M_{tIII} - 3M + M + M_e = 0 \Rightarrow$$

$$M_{tIII} = 3M - M - M_e = M.$$

calcul de dimensions.

$$\tau_{max} = \frac{Mt_{max}}{W_P} \leq [\tau].$$

calcul de d1.

$$W_{PI} \geq \frac{Mt_I max}{[\tau]} \qquad W_{PI} = \frac{\pi d_1^3}{16} \geq \frac{Mt_1 max}{[\tau]}$$

$$\Rightarrow d_1 \geq \sqrt[3]{\frac{16\ Mt_1 max}{\pi\ [\tau]}} \geq \sqrt[3]{\frac{16\ |-200|}{\pi \cdot 40 \cdot 10^6}} \geq 2,942 \cdot 10^{-2}\ m$$

$d_1 \geq 29,4$ mm. on choisit $d_1 = 30$ mm.

Calcul de d_2

$$W_{PII} \geq \frac{Mt_{II} max}{[\tau]} \qquad W_{PII} = \frac{\pi d_{II}^3}{16} \geq \frac{Mt_{II} max}{[\tau]}$$

$$\Rightarrow d_{II} \geq \sqrt[3]{\frac{16\ Mt_{II} max}{\pi\ [\tau]}} \geq \sqrt{\frac{16 \cdot |-400|}{\pi \cdot 40 \cdot 10^6}}$$

$d_2 \geq 37,06$ mm on choisit $d_2 = 38$ mm.

calcul de d_3

$W_{P_{III}} \geqslant \frac{Mt_{III}\,max}{[\tau]}$ comme $|Mt_{III}\,max| = |Mt_{I}\,max|$

$\Rightarrow d_1 = d_2 = 29,4$ mm on choisit $d_3 = 30$ mm

calcul des angles de torsions.

$$\alpha_I = \int_0^{0,4} \frac{Mt_1\,dx_1}{GI_{o_1}} = \frac{1}{GI_{o_1}} \int_0^{0,4} -M\,dx_1 = \frac{-1}{GI_{o_1}} M X_1 \Big|_0^{0,4}$$

pour $x_1 = 0 \Rightarrow \alpha_I = 0$.

$x_1 = 0,4m \Rightarrow \alpha_I = \frac{-1}{GI_{o_1}} 0,4\,M$.

$$\alpha_{II} = \frac{1}{GI_{o_2}} \int_0^{0,4} Mt_{II}\,dx_2 = -\frac{1}{GI_{o_2}} 2M X_2 \Big|_0^{0,4}$$

pour $X_2 = 0 \Rightarrow \alpha_{II} = 0$.

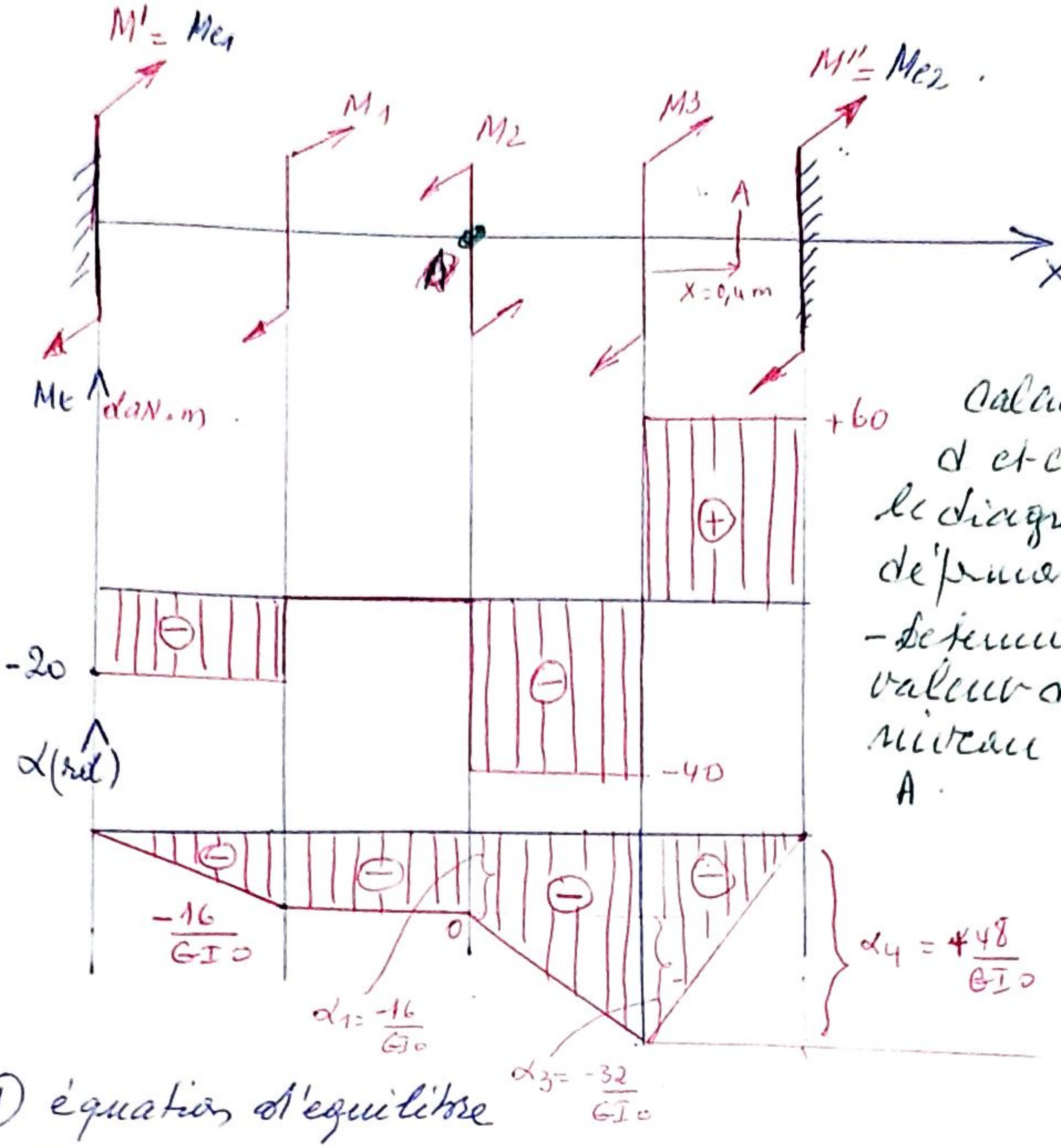
$X_2 = 0,4 \Rightarrow \alpha_{II} = \frac{-0,8\,M}{GI_{o_2}}$.

$$\alpha_{III} = +\frac{1}{GI_{o_3}} \int_0^{0,4} Mt_{III}\,dx_3 = \frac{1}{GI_{o_3}} M x_3 \Big|_0^{0,4}$$

pour $X_3 = 0 \Rightarrow \alpha_{III} = 0$

$X_3 = 0,4 \Rightarrow \alpha_{III} = \frac{0,4\,M}{GI_{o_3}}$.

$\alpha_I + \alpha_{II} + \alpha_{III} = \alpha_{total}$. $\alpha_{tot} > 0$



Calculer le diamètre d et construire le diagramme de déformation α
- Déterminer la valeur de α au niveau de la section A.

(1) équation d'équilibre

$$\Sigma M/x = 0 \Rightarrow -M'' - M_3 + M_2 - M_1 + M' = 0 \quad (1)$$

$$M'' = M_2 - M_3 - M_1 - M' = 40 - 100 - 20 - (-20)$$

1eq - 2 in = 1er degré hyperstatisme

$$M'' = -60$$

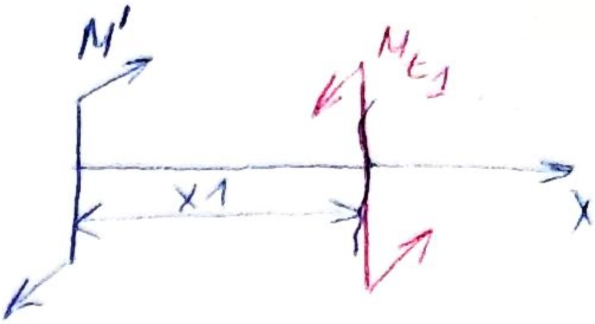
(2) équation de compatibilité ou de déplacement angulaire ou déformation angulaire.

$$\alpha_{total} = \sum_{i=1}^{n=4} \alpha_i = 0 \quad (2)$$

données: $M_1 = 20$ daN.m, $M_2 = 40$ daN.m, $M_3 = 100$ daN.m

$G = 8.10^5$ daN/cm², $[\tau] = 500$ daN/cm², $l = 0,8$ m.

Tronçon I $\quad 0 \leq x_1 \leq 0{,}8\,m$



$\Sigma M/x_I = 0 = M_{t_1} - M' = 0$

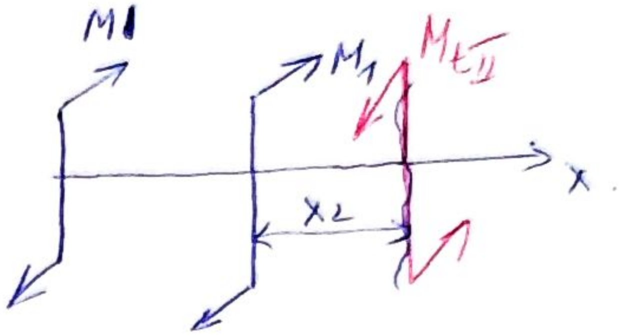
$M_{t_1} = M' = -20\,daN.m$

$$\alpha_1 = \int_0^{0,8} \frac{M_{t_1}\,dx_1}{GI_0} = \left.\frac{M' x_1}{GI_0}\right|_0^{0,8}$$

pour $x_1 = 0 \qquad \alpha_1 = 0$

$x_1 = 0{,}8 \qquad \alpha_1 = \dfrac{0{,}8\,M'}{GI_0} = \dfrac{0{,}8\cdot(-20)}{GI_0} = \dfrac{-16}{GI_0}$

Tronçon II $\quad 0 \leq x_2 \leq 0{,}8$



$\Sigma M/x = 0 = M_{t_{II}} - M_1 - M' = 0$

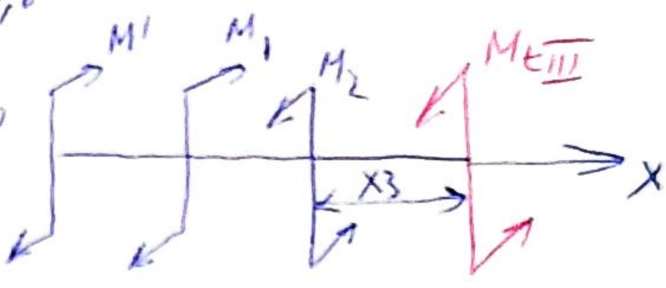
$M_{t_{II}} = M_1 + M' = 20 - 20 = 0$

$$\alpha_2 = \int_0^{0,8} \frac{(M_1 + M')\,dx_2}{GI_0} = \left.\frac{(M_1 + M')\,x_2}{GI_0}\right|_0^{0,8}$$

pour $x_2 = 0 \qquad \alpha_2 = 0$.

$x_2 = 0{,}8 \qquad \alpha_2 = \dfrac{(M_1 + M')\,0{,}8}{GI_0} = \dfrac{(20 - 20)\,0{,}8}{GI_0} = 0$

Tronçon III $\quad 0 \leq x_3 \leq 0{,}8$



$\Sigma M/x = 0 : M_{t_{III}} + M_2 - M_1 - M' = 0$

$M_{t_{III}} = M_1 + M' - M_2$

$M_{t_{III}} = 20 - 20 - 40 = -40\,daN.m$

(3)

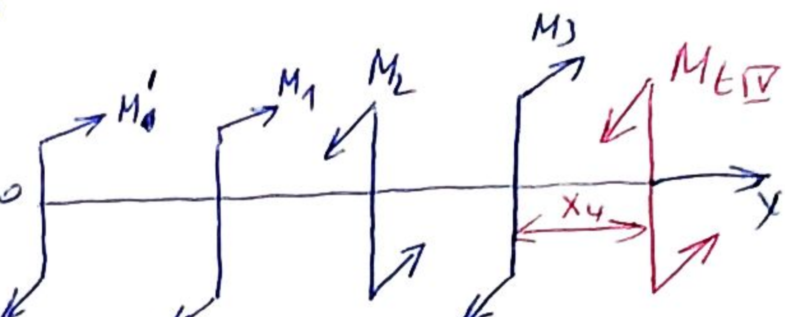
$$\alpha_3 = \int_0^{0,8} \frac{M_{t_{III}}\, dx_3}{GI_0} = \int_0^{0,8} \frac{(M_1 + M' - M_2)\, dx_3}{GI_0} = \frac{(M_1 + M' - M_2) X_2}{GI_0} \Big|_0^{0,8}$$

pour $X_3 = 0 \qquad \alpha_3 = 0$

$X_3 = 0,8 \qquad \alpha_3 = \dfrac{(M_1 + M' - M_2)\, 0,8}{GI_0} = \dfrac{(\cancel{20} - \cancel{20} - 40)\, 0,8}{GI_0}$

$\alpha_3 = \dfrac{-32}{GI_0}$

Tronçon $\underline{IV}$ $\quad 0 \leq x_4 \leq 0,8$



$\Sigma M/x = 0 \quad M_{t_{IV}} - M_3 + M_2 - M_1 - M' = 0$

$M_{t_{IV}} = M_3 - M_2 + M_1 + M'$

$M_{t_{IV}} = 100 - 40 + 20 - 20 = 60$ daN.m.

$$\alpha_4 = \int_0^{0,8} \frac{M_{t_{IV}}\, dx_4}{GI_0} = \int_0^{0,8} \frac{(M_3 - M_2 + M_1 + M')\, dx_4}{GI_0} =$$

$$= \frac{(M_3 - M_2 + M_1 + M') X_4}{GI_0} \Big|_0^{0,8}$$

pour $X_4 = 0 \qquad \alpha_4 = 0$

$X_4 = 0,8 \qquad \alpha_4 = \dfrac{(M_3 - M_2 + M_1 + M')\, 0,8}{GI_0}$

$\alpha_4 = \dfrac{(100 - 40 + \cancel{20} - \cancel{20})\, 0,8}{GI_0}$

Revenons à l'équation (2)

$\alpha_4 = \dfrac{48}{GI_0}$

$$\sum_{i=1}^{n=4} \alpha_i = 0 = \frac{0,8\, M'}{GI_0} + \frac{(M' + M_1)\, 0,8}{GI_0} + \frac{(M' + M_1 - M_2)\, 0,8}{GI_0}$$

$$+ \frac{(M' + M_1 - M_2 + M_3)\, 0,8}{GI_0} = 0.$$

④

$$M' + M' + M_1 + M' + M_1 - M_2 + M' + M_1 - M_2 + M_3 = 0$$

$$4M' + 3M_1 - 2M_2 + M_3 = 0$$

$$M' = \frac{2M_2 - 3M_1 - M_3}{4} = \frac{2.40 - 3.20 - 100}{4}$$

$$M' = \frac{80 - 60 - 100}{4} = \frac{-80}{4} = \boxed{-20 \text{ daNm}}$$

Déterminer α_A

$$\alpha_A = \alpha_1 + \alpha_2 + \alpha_3 + \int_0^{0,4} \frac{(M_{f4})}{GI_0} dx_4$$

$$\alpha_A = -\frac{16}{GI_0} + 0 - \frac{32}{GI_0} + \frac{(M_3 - M_2 + M_1 + M')x_4}{GI_0} \Big|_0^{0,4}$$

pour $x_4 = 0,4$ $\quad \alpha_4 = \frac{(100 - 40 + 20 - 20)0,4}{GI_0} = 24/GI_0$

$$\alpha_A = -\frac{16}{GI_0} + 0 - \frac{32}{GI_0} + \frac{24}{GI_0} = \frac{-24}{GI_0}$$

autre méthode

$$\alpha_A = \int_0^{0,4} \frac{+M'' dx}{GI_0} = -\frac{M''x}{GI_0}\Big|_0^{0,4} = \frac{-(-60)x\Big|_0^{0,4}}{GI_0}$$

$$\alpha_A = \frac{60x}{GI_0}\Big|_0^{0,4}$$

pour $x = 0 \quad \alpha_A = 0$

$x = 0,4 \quad \alpha_A = \frac{-24}{GI_0}$