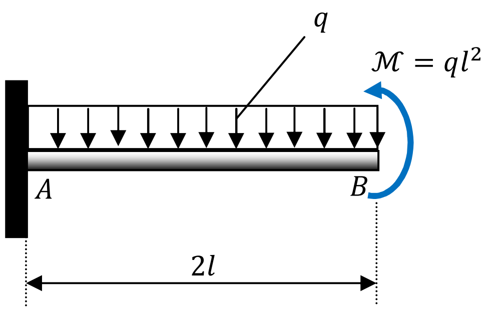
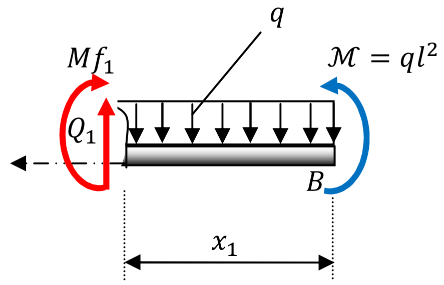
Exercice N° 1:





A:

1) Construire les diagrammes de Q et Mf:

Tronçon I: $0 \leq x_1 \leq 2l$

$$\sum Fext_{/Y} = 0 \implies Q_1 - qx_1 = 0$$

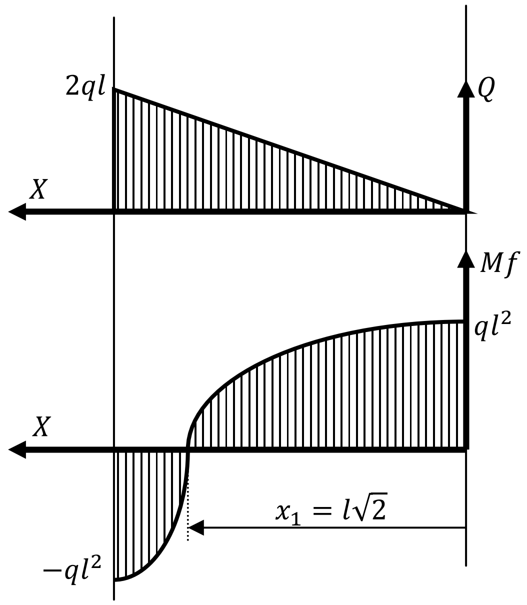
$$\begin{cases} x_1 = 0 \implies Q_1 = 0 \\ x_1 = 2l \implies Q_1 = 2ql \end{cases}$$

$$\sum MFext_{/Z} = 0 \implies \mathcal{M} - q\frac{x_1^2}{2} - Mf_1 = 0$$

$$Mf_1 = \mathcal{M} - q\frac{x_1^2}{2}$$

$$\begin{cases} x_1 = 0 \implies Mf_1 = +ql^2 \\ x_1 = 2l \implies Mf_1 = -ql^2 \end{cases}$$

$$Mf_1 = ql^2 - q\frac{x_1^2}{2} = 0 \implies x_1 = l\sqrt{2}$$



2) Déterminer la position du centre de gravité:

OZ : axe de symétrie ==> $Z_G = 0$

$$Y_G = \frac{\sum S_{Zi}}{\sum S_i} = \frac{6a(144a^2) - 4a(72a^2)}{144a^2 - 72a^2} = 8a$$

3) construction des diagrammes de σ et τ:

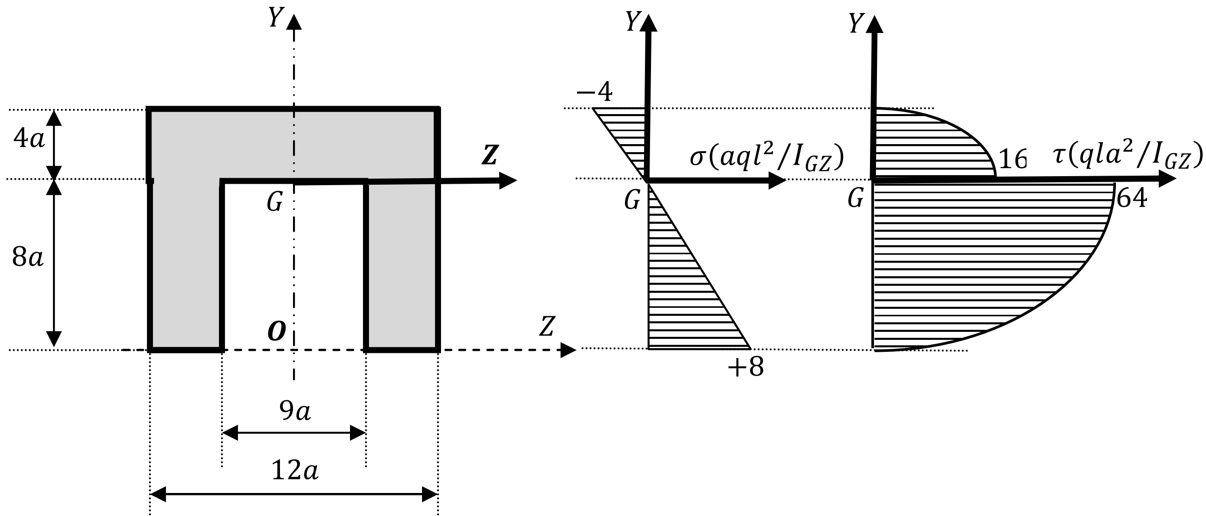
Des diagrammes de Q et Mf, on tire: $Q_{max} = 2ql$, $Mf_{max} = +ql^2$

a) $\sigma = -\frac{Mf_{max}}{I_{GZ}}.y$

$$\begin{cases} y = 0 \implies \sigma = 0 \\ y = 4a \implies \sigma = -4ql^2/I_{GZ} \\ y = -8a \implies \sigma = +8ql^2/I_{GZ} \end{cases}$$

b) $\tau = \frac{|Q_{max}||S^*_{GZ}|}{I_{GZ}.b_y}$

$$\begin{cases} y = 4a \implies \tau_1 = 0 \\ y = -8a \implies \tau_1 = 0 \\ y = 0 \implies S^* = (12a.4a)2a = 96a^3 \begin{cases} b = 12a \implies \tau = \dfrac{2ql}{I_{GZ}}\dfrac{96a^3}{12a} = \dfrac{16qla^2}{I_{GZ}} \\ b = 3a \implies \tau = \dfrac{2ql}{I_{GZ}}\dfrac{96a^3}{12a} = \dfrac{64qla^2}{I_{GZ}} \end{cases} \end{cases}$$



4) Déterminer la position des sections dans lesquelles on a:

a) Flexion pure: $Mf \neq 0,\ Q = 0$ ==> Section située à $x = 0$

b) cisaillement pur: $Mf = 0,\ Q \neq 0$ ==> Section située à $x = l\sqrt{2}$

B :

1) Déterminer le moment d'inertie I_{GZ} de la section droite puis calculer " a " pour que la pièce résiste.

a) Calcul du moment d'inertie:

$$I_{GZ} = \left[\frac{12a(4a)^3}{3}\right] + 2\left[\frac{1{,}5a(8a)^3}{12} + (8a.1{,}5a)16a^2\right] = 768a^4$$

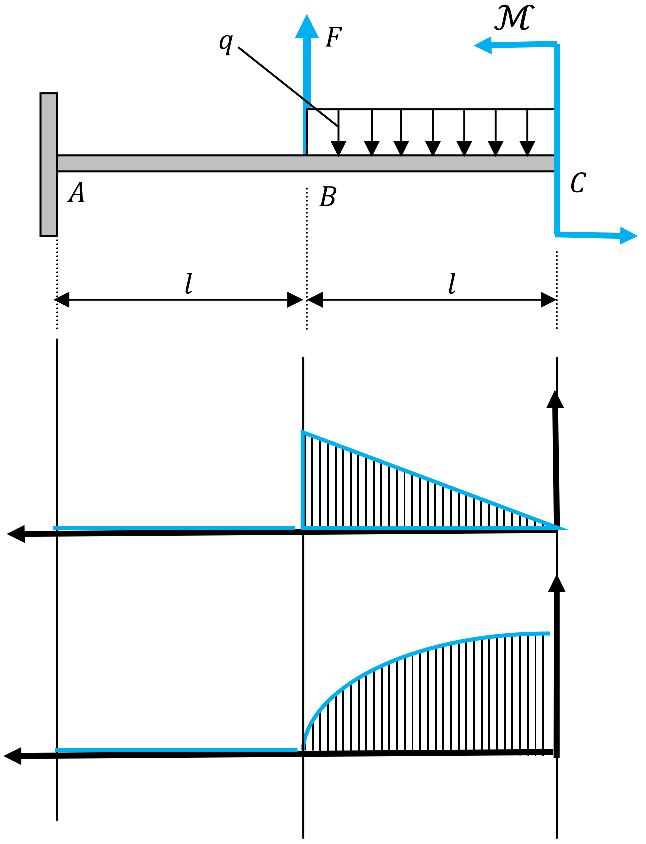
b) Calcul de " a " pour que la pièce résiste: $I_{GZ} = 768a^4$

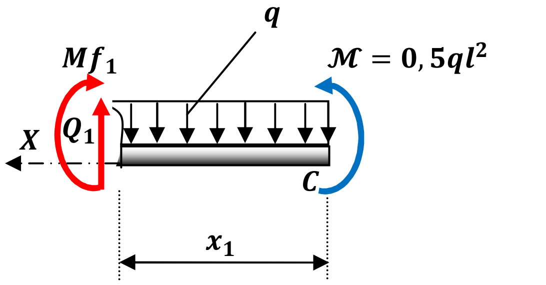
$$\left.\begin{array}{l} a^* \ \sigma_{max} = \dfrac{ql^2.8a}{768a^4} \leq [\sigma] \implies a = \sqrt[3]{\dfrac{ql^2}{96[\sigma]}} = \sqrt[3]{\dfrac{200.100^2}{96.200}} = 4{,}70\ (cm) \\ b^* \ \tau_{max} = \dfrac{64qla^2}{768a^4} \leq [\tau] \implies a = \sqrt[2]{\dfrac{ql}{12[\tau]}} = \sqrt[3]{\dfrac{200.100}{12.40}} = 6{,}65\ (cm) \end{array}\right\} \text{on choisi } a = 6{,}5\ cm$$

2) Application numérique: Remplir le tableau suivant

$\lvert Q_{max}\rvert$	Mf_{max}	a	I_{GZ}	σ_{max}	τ_{max}
(N)	$(N.m)$	(cm)	(cm^4)	(N/cm^2)	(N/cm^2)
4.10^4	200.10^4	6,5	1370928	75,86	39,45

Exercice N° 2:





1) Supposer $\boldsymbol{l, a, F = ql, \mathcal{M} = 0,5ql^2}$ **et** $\boldsymbol{I_{GZ}}$ connus:

a) Construire les diagrammes de $\boldsymbol{Q}$ et $\boldsymbol{Mf}$:

Tronçon I: $\boldsymbol{0 \leq x_1 \leq l}$

$$\sum Fext_{/Y} = 0 \implies Q_1 - qx_1 = 0$$

$$\begin{cases} x_1 = 0 \implies Q_1 = 0 \\ x_1 = l \implies Q_1 = ql \end{cases}$$

$$\sum MFext_{/Z} = 0 \implies \mathcal{M} - q\frac{x_1^2}{2} - Mf_1 = 0$$

$$Mf_1 = \mathcal{M} - q\frac{x_1^2}{2}$$

$$\begin{cases} x_1 = 0 \implies Mf_1 = +0,5ql^2 \\ \quad x_1 = l \implies Mf_1 = 0 \end{cases}$$

Tronçon **II**: $\boldsymbol{0 \leq x_2 \leq l}$

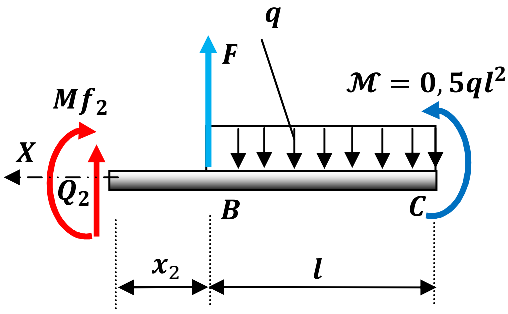
$$\sum \boldsymbol{Fext_{/Y} = 0 \implies Q_2 + F - ql = 0}$$

$$Q_2 = \boldsymbol{F} - ql = \boldsymbol{0}$$

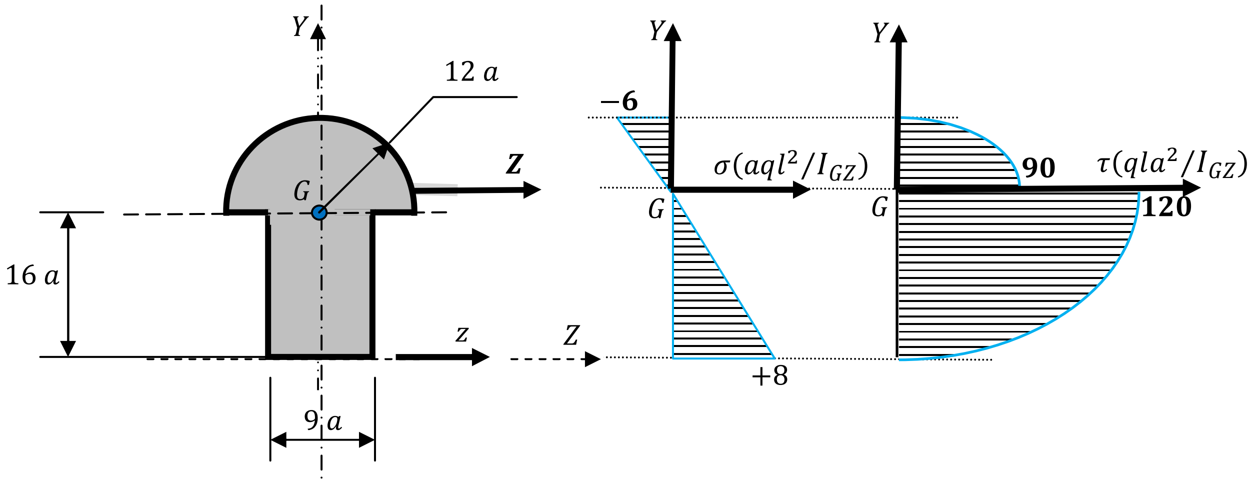
$$\sum MFext_{/Z} = 0 \implies \mathcal{M} - q\frac{x_1^2}{2} - Mf_1 = 0$$

$$Mf_1 = \mathcal{M} - q\frac{x_1^2}{2}$$

$$\begin{cases} x_2 = 0 \implies Mf_2 = \boldsymbol{0} \\ x_2 = l \implies Mf_2 = 0 \end{cases}$$



b) construction des diagrammes des contraintes normales $\boldsymbol{\sigma}$ et tangentielles $\boldsymbol{\tau}$ agissant dans les sections dangereuses.
Section dangereuse **C** ou agit $\boldsymbol{Mf_{max} = 0,5ql^2}$
Section dangereuse **B** ou agit $\boldsymbol{Q = ql}$



a) $\sigma = -\frac{Mf_{max}}{I_{GZ}}.y$

$$\begin{cases} y = 0 \implies \sigma = 0 \\ y = 12a \implies \sigma = -0,5ql^2(12a)/I_{GZ} = -6aql^2/I_{GZ} \\ y = -8a \implies \sigma = -0,5ql^2(-16a)/I_{GZ} = 8aql^2/I_{GZ} \end{cases}$$

b) $\tau = \frac{|Q_{max}||S^*_{GZ}|}{I_{GZ}.b_y}$

$$\begin{cases} y = 12a \implies \tau_1 = 0 \\ y = -16a \implies \tau_1 = 0 \\ y = 0 \implies S^* = (9a.16a)8a = 1080a^3 \begin{cases} b = 12a \implies \tau = \frac{ql}{I_{GZ}}\frac{1080a^3}{12a} = \frac{90qla^2}{I_{GZ}} \\ b = 3a \implies \tau = \frac{ql}{I_{GZ}}\frac{1080a^3}{9a} = \frac{120qla^2}{I_{GZ}} \end{cases} \end{cases}$$

C) Déterminer la position des sections dans lesquelles on a:

a) Flexion pure: $Mf \neq 0, Q = 0$ ==> Section située à $x_1 = 0$ (Section C)
b) cisaillement pur: $Mf = 0, Q \neq 0$ ==> Section située à $x = l$ (Section B)

2) Prendre a 1 cm
a) Détermination des coordonnées du centre de gravité:
OZ : axe de symétrie ==> $Z_G = 0$

$$Y_G = \frac{\sum S_{Zi}}{\sum S_i} = \frac{8(9.16) + \left(16 + \frac{4.12}{3\pi}\right)\left(\frac{\pi 12^3}{2}\right)}{(9.16) + \left(\frac{\pi 12^3}{2}\right)} = 16\ (cm)$$

b) Calcul du moment d'inertie I_{GZ}:

$$I_{GZ} = I_{1GZ} + I_{2GZ} = \frac{9.16^3}{12} + 8^2(8.16) + \frac{\pi 12^4}{8} = 20429,47\ (cm^4)$$