SOLUTION

Exercice N° 1:

La contrainte en chaque point est définie par: $\tau = G\theta\rho$

elle est maximum pour $\rho_{max} = D/2 = R$

Donc $\tau_{max} = G\theta\, D/2$

La condition de résistance est : $\tau_{max} = G\theta\, D/2 \leq R_p$

On obtient le diamètre de la forme suivant:

$$D \leq 2\frac{R_p}{G\theta} = \frac{2.400}{8.10^4\ 0{,}29.10^{-3}} = 34{,}48\ mm$$

d'ou on calcul : $\theta = \frac{\alpha}{l} = \frac{20}{1200}.\frac{\pi}{180} = 0{,}29.10^{-3}\frac{rd}{mm}$

on choisit le diamètre $D = 34\ mm$

L'équation du moment de torsion est: $M_t = G\theta I_0 = G\theta\frac{\pi}{32}(D^4 - d^4)$

On tire $d = \sqrt[4]{D^4 \frac{32M_t}{G\theta\pi}}$

Calculons d'abord le moment de torsion M_t

$$M_t = \frac{P}{\omega} = \frac{P}{\pi n/30} = \frac{30.314.10^3}{\pi.1500} = 2000\ Nm = 2.10^{6\ N.mm}$$

Les unités doivent être
$P = (Watt)$, $n = (tr/min)$
On tire $M_t = (N.m)$

Donc $d = \sqrt[4]{34^4 \frac{32.2.10^6}{8.10^4.0{,}29.10^{-3}.\pi}} = 26\ mm$

Vérification:

1) Condition de résistance : $\tau_{max} \leq R_p = 400\ \ N/mm^2$

$$\tau_{max} = \frac{M_t}{I_0}.\frac{D}{2} = \frac{2.10^6.17}{\frac{\pi}{32}(34^4 - 26^4)} = 393{,}84\ N/mm^2 < 400\ N/mm^2$$

2) Condition de rigidité : $\alpha \leq 20^o$

$$\alpha = \frac{M_t l}{G I_0}.\frac{180}{\pi} = \frac{2.10^6.1200.180}{8.10^4\frac{\pi}{32}(34^4 - 26^4)\pi} = 19{,}91^o < 20^o$$

Les deux conditions sont vérifiées pour
$D = 34\ mm$ et $d = 26\ mm$.

Exercice N° 2:

a) Construire les diagrammes du moment de torsion Mt, de l'angle de torsion α et déterminer la valeur de la contrainte tangentielle maximale τ_{max}.

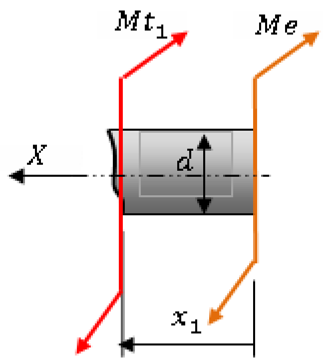
1. Equation d'équilibre:

$$\sum M_t = Me + 2M + M = 0$$

$$M_e = -3M$$

a. Tronçon I $\quad 0 \le x_1 \le l$

$$Mt_1 + Me = 0 \implies Mt_1 = +3M$$



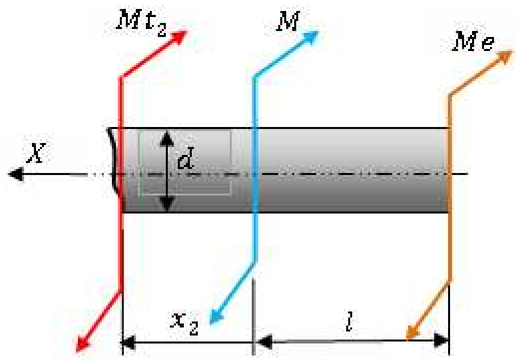
$$\alpha_1 = \int_0^{x_1} \frac{Mt_1 dx_1}{GI_0} = \frac{3Mx_1}{GI_0}$$

$$\begin{cases} pour\ x_1 = 0 \implies \alpha_1 = 0 \\ pour\ x_1 = l \implies \alpha_1 = \dfrac{3Ml}{GI_0} \end{cases}$$

b. Tronçon II $\quad 0 \le x_1 \le l$

$$Mt_2 + Me + M = 0$$

$$Mt_2 = -Me - M = -3M - M = +2M$$



$$\alpha_2 = \int_0^{x_2} \frac{Mt_2 dx_2}{GI_0} = \frac{2Mx_2}{GI_0}$$

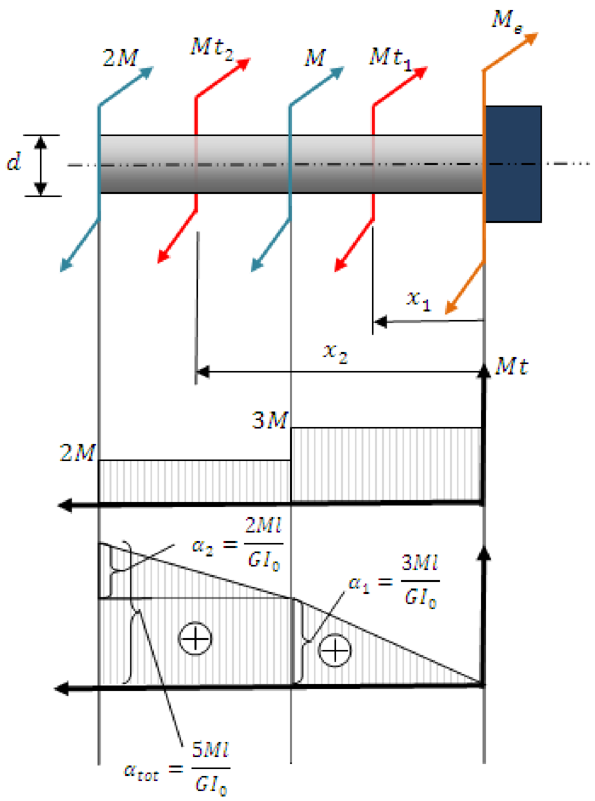
$$\begin{cases} pour\ x_2 = 0 \implies \alpha_2 = 0 \\ pour\ x_2 = l \implies \alpha_2 = \dfrac{2Ml}{GI_0} \end{cases}$$

Angle de torsion totale à l'extrémité

$$\alpha_{tot} = \alpha_1 + \alpha_2 = \frac{3Ml}{GI_0} + \frac{2Ml}{GI_0} = \frac{5Ml}{GI_0}$$

La contrainte tangentielle maximum est: $\tau_{max} = \frac{Mt_{max}}{W_p}$

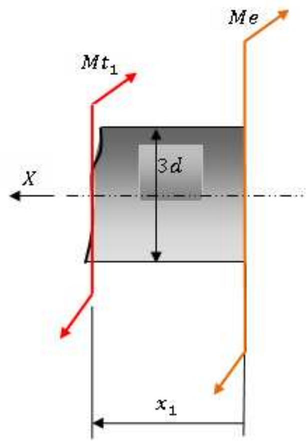
D'où $W_p = \frac{I_0}{v} = \frac{\pi d^4/32}{d/2} = \frac{\pi d^3}{16}$ ce qui donne $\tau_{max} = \frac{Mt_{max}}{W_p} = \frac{16.3M}{\pi d^3}$



Exercice N° 2:

b) Construire les diagrammes du moment de torsion M_t, de l'angle de torsion α et déterminer la valeur de la contrainte tangentielle maximale.

1. Equation d'équilibre:



$$\sum M_t = Me - M - 3M + M = 0$$

$$Me = 3M$$

a. Tronçon I $\qquad 0 \leq x_1 \leq 1.5\, l$

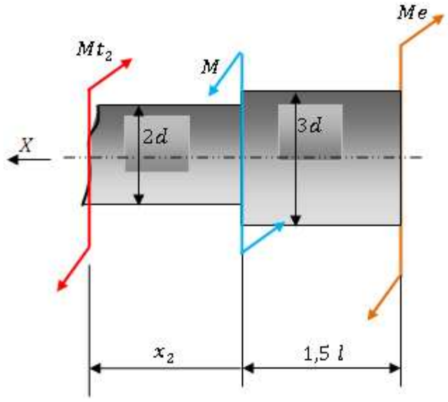
$$Mt_1 + Me = 0$$

$$Mt_1 = -Me = -3M$$

$$\alpha_1 = \int_0^{x_1} \frac{Mt_1 dx_1}{GI_0} = \frac{-3Mx_1}{GI_{01}}$$

$$\begin{cases} pour\ x_1 = 0 \ ==> \ \alpha_1 = 0 \\ pour\ x_1 = 1{,}5\ l \ ==> \ \alpha_1 = -\dfrac{4{,}5Ml}{81GI_0} = -0{,}055\dfrac{Ml}{GI_0} \end{cases}$$

b. Tronçon II $\quad 0 \leq x_1 \leq 1{,}5\ l$



$$Mt_2 + Me - M = 0$$

$$Mt_2 = -Me = -2M$$

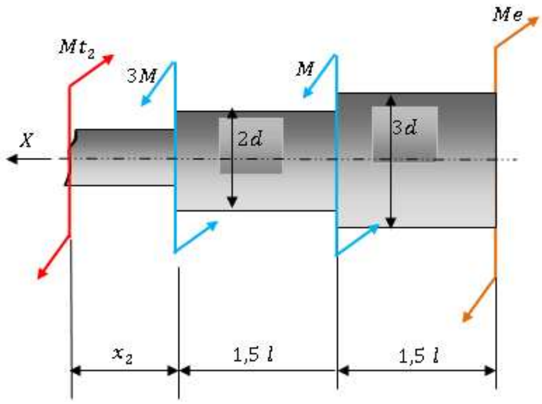
$$\alpha_2 = \int_0^{x_2} \frac{Mt_2 dx_2}{GI_{02}} = -\frac{2Mx_2}{GI_{02}}$$

$$\begin{cases} pour\ x_2 = 0 \ ==> \ \alpha_2 = 0 \\ pour\ x_2 = 1{,}5\ l \ ==> \ \alpha_2 = -\dfrac{3Ml}{16GI_0} = -0{,}1875\dfrac{Ml}{GI_0} \end{cases}$$

b. Tronçon III $\quad 0 \leq x_3 \leq l$

$$Mt_3 + Me - M - 3M = 0$$

$$Mt_3 = -Me + M + 3M = M$$



$$\alpha_3 = \int_0^{x_3} \frac{Mt_3 dx_3}{GI_{03}} = \frac{Mx_2}{GI_{03}}$$

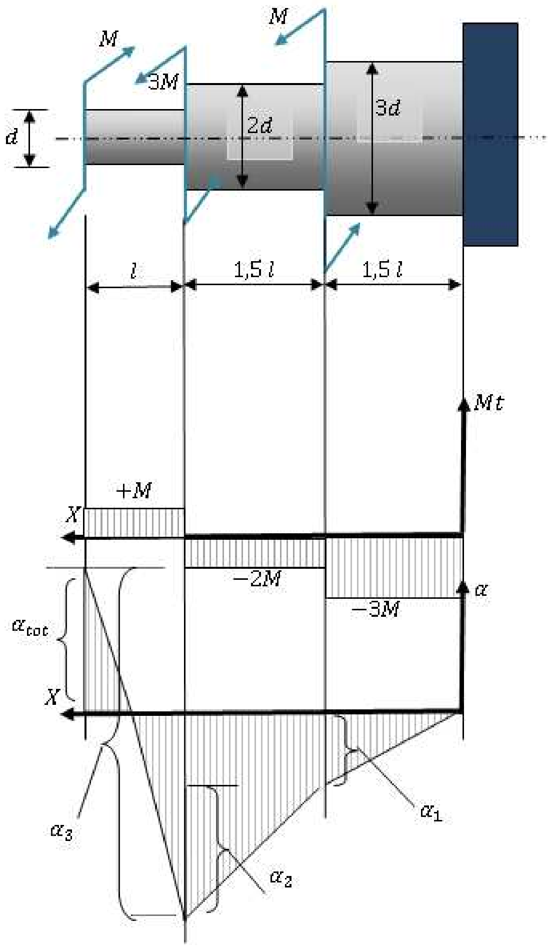
$$\begin{cases} pour\ x_3 = 0 \ ==> \ \alpha_3 = 0 \\ pour\ x_3 = l \ ==> \ \alpha_3 = -\dfrac{3Ml}{16GI_0} = \dfrac{Ml}{GI_0} \end{cases}$$

Angle de torsion totale à l'extrémité

$$\alpha_{tot} = \alpha_1 + \alpha_2 + \alpha_3 = -\frac{0{,}055}{GI_0} - \frac{0{,}01875Ml}{GI_0} - \frac{Ml}{GI_0} = \frac{0{,}7575Ml}{GI_0}$$

La contrainte tangentielle maximum est:

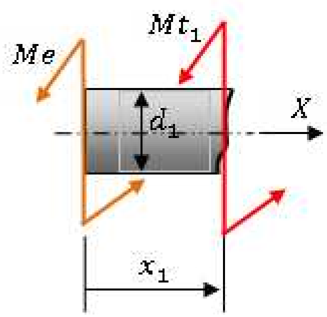
D'où $\quad W_{p1} = \frac{I_{01}}{v} = \frac{\pi 81 d^4/32}{3d/2} = \frac{\pi 27 d^3}{16}$ $\quad$ ce qui donne $\tau_{max} = \frac{Mt_{max}}{W_{p1}} = -\frac{48M}{27\pi d^3}$



Exercice N° 3:
Déterminer les dimensions des sections droites des barres assurant leurs résistances et calculer leurs angles de torsion.
1. Equation d'équilibre:

$$\sum M_t = Me + M - M + M = 0$$

$Me = M$



a. Tronçon I $\qquad 0 \leq x_1 \leq l$

$Mt_1 + Me = 0 \implies Mt_1 = -Me = -M$

$\tau_1 = \frac{Mt_1}{W_{p1}} \leq [\tau] \qquad W_{p1} = \frac{\pi d_1^3}{16}$

$$d_1 = \sqrt[3]{\frac{16M}{\pi[\tau]}} = \sqrt[3]{\frac{16.200}{\pi.40.10^6}} = 2{,}942.10^{-2}\ m$$

$$I_{01} = \frac{\pi d^4}{32} = 7{,}354.10^{-8}\, m^4$$

$$\alpha_1 = \int_0^{x_1} \frac{Mt_1 dx_1}{GI_{01}} = \frac{-Mx_1}{GI_{01}}$$

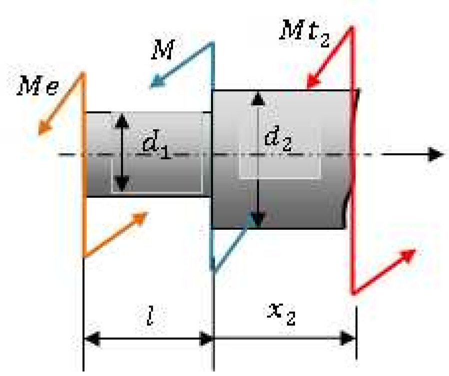
$$\begin{cases} pour\ x_1 = 0\ \ ==>\ \ \alpha_1 = 0 \\ pour\ x_1 = l\ \ ==>\ \ \alpha_1 = -\dfrac{Ml}{GI_{01}} = -\dfrac{200.0{,}4}{8.10^{10}7{,}354.10^{-8}} = -0{,}0136\, rd \end{cases}$$

a. Tronçon II $\qquad 0 \le x_2 \le l$

$$Mt_2 + Me + M = 0\ \ ==>\ \ Mt_{12} = -Me - M = -2M$$

$$\tau_2 = \frac{Mt_2}{W_{p2}} \le [\tau] \qquad\qquad W_{p2} = \frac{\pi d_2^3}{16}$$

$$d_2 = \sqrt[3]{\frac{16.2M}{\pi[\tau]}} = \sqrt[3]{\frac{16.400}{\pi.40.10^6}} = 3{,}706.10^{-2}\, m$$



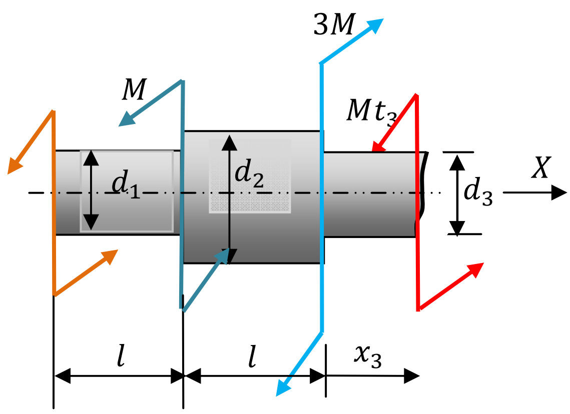
$$I_{02} = \frac{\pi d_2^4}{32} = \frac{\pi(3{,}706.10^{-2})^4}{32} = 1{,}85.10^{-7}\, m^4$$

$$\alpha_2 = \int_0^{x_2} \frac{Mt_2 dx_2}{GI_{01}} = \frac{-2Mx_2}{GI_{02}}$$

$$\begin{cases} pour\ x_1 = 0\ \ ==>\ \ \alpha_1 = 0 \\ pour\ x_2 = l\ \ ==>\ \ \alpha_2 = -\dfrac{Ml}{GI_{01}} = -\dfrac{400.0{,}4}{8.10^{10}1{,}85.10^{-7}} = -0{,}0108\, rd \end{cases}$$

a. Tronçon III $\qquad 0 \le x_3 \le l$

$$Mt_3 + M_e + M - 3M = 0\ \ ==>\ \ Mt_3 = M$$



$$\tau_3 = \frac{Mt_3}{W_{p3}} \le [\tau] \qquad\qquad W_{p3} = \frac{\pi d_3^3}{16}$$

$$d_3 = \sqrt[3]{\frac{16.M}{\pi[\tau]}} = \sqrt[3]{\frac{16.200}{\pi.40.10^6}} = 2{,}942.10^{-2}\, m$$

$$I_{03} = \frac{\pi d_3^4}{32} = \frac{\pi(2{,}942.10^{-2})^4}{32} = 7{,}354.10^{-8}\, m^4$$

$$\alpha_3 = \int_0^{x_3} \frac{Mt_3 dx_3}{GI_{03}} = \frac{Mx_3}{GI_{03}}$$

$$\begin{cases} pour\ x_3 = 0 \ ==> \ \alpha_3 = 0 \\ pour\ x_3 = l \ ==> \ \alpha_3 = \dfrac{Ml}{GI_{03}} = -\dfrac{200.0,4}{8.10^{10}7,354.10^{-8}} = 0,0136\ rd \end{cases}$$