Solution Examen MMC

Question de cours

① $B(X,t) = b(\Phi(X,t),t)$, $b(x,t)$ 　 X : variable de Lagrange, x = " d'Euler

$$\frac{db}{dt} = \frac{\partial B}{\partial t} + \operatorname{grad} b \,.\, V.$$

$$\operatorname{grad} b \,.\, V = \sum_{i=1}^{3} \frac{\partial b}{\partial x_i} \frac{\partial \phi_i}{\partial X}$$

② $C(t) = \int_{D_0} c^{(L)}(X,t)\, J(X,t)\, dX \Rightarrow$

$$C'(t) = \int_{D_0} \left[\frac{\partial c^{(L)}}{\partial t} J(X,t) + c^L(X,t) \frac{\partial J}{\partial t}(X,t) \right] dX$$

$$= \int_{D_0} \left[\frac{\partial c^{(L)}}{\partial t} + c^L(X,t)\,(\operatorname{div} V) \right] J(X,t)\, dX$$

$$= \int_{D(t)} \left[\frac{\partial c}{\partial t} + \operatorname{grad} c\, V + c \operatorname{div} V \right] dx$$

$$= \int_{D(t)} \left[\frac{dc}{dt} + c \operatorname{div} V \right] dx$$

Exercice

① $F(t) = \begin{pmatrix} 1 & 2\alpha t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $C(t) = \begin{pmatrix} 1 & 2\alpha t & 0 \\ 2\alpha t & 1+4\alpha^2 t^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $\varepsilon(t) = \begin{pmatrix} 0 & \alpha t & 0 \\ \alpha t & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\varepsilon(t) = \dfrac{\nabla u + \nabla u^T}{2}$, $\nabla u = \left(\dfrac{\partial u_i}{\partial x_j} \right)$, $u_i = x_i - X_i = \phi_i(X,t) - X_i$

$u_1 = 2\alpha t X_2$, $u_2 = u_3 = 0$

② $V^{(L)}(X,t) = \dfrac{\partial \phi}{\partial t} = (2\alpha X_2, 0, 0)$

$X_2 = x_2 \Rightarrow V^{(E)}(x,t) = (2\alpha x_2, 0, 0)$

$K = \operatorname{grad} V = \left(\dfrac{\partial V_i}{\partial x_j} \right) = \begin{pmatrix} 0 & 2\alpha & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $D = \frac{1}{2}(K + {}^tK)$

$D = \begin{pmatrix} 0 & \alpha & 0 \\ \alpha & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$D_{ii} = 0$ taux d'allongement dans la direction e_i

Si $\delta x = \lambda(t)\, e_i \Rightarrow T_i = \dfrac{1}{\|\delta x\|} \dfrac{d}{dt} \|\delta x(t)\| = \dfrac{(D\delta x, \delta x)}{\|\delta x\|^2}$