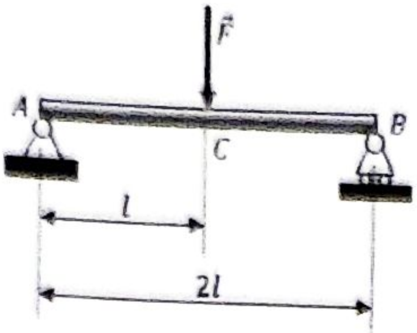
S.T 2ème Année — TD Série N° 5.1 — RDM — Université Badji-Mokhtar

Exercice : Soit les figures suivantes:

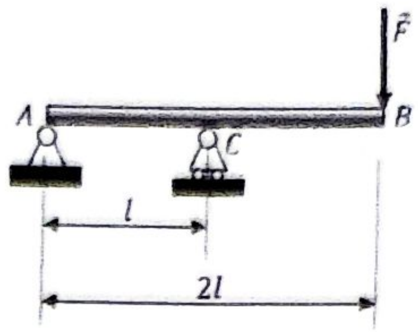
1- Tracer les diagrammes de l'effort tranchant et du moment fléchissant.

2- Déterminer les sections où agissent la flexion pure et le cisaillement pur.

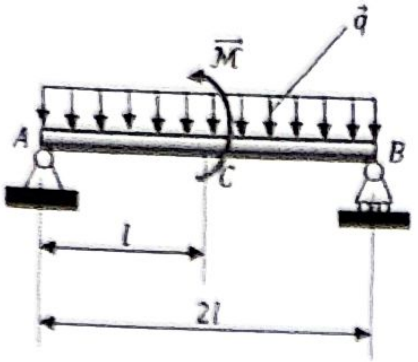
Données : $\mathcal{M} = ql^2,\ q = \frac{2F}{l},\ F,\ l$



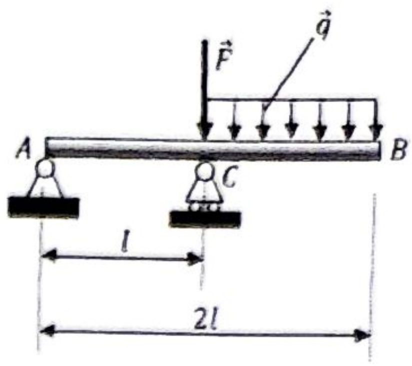
Ex. 1



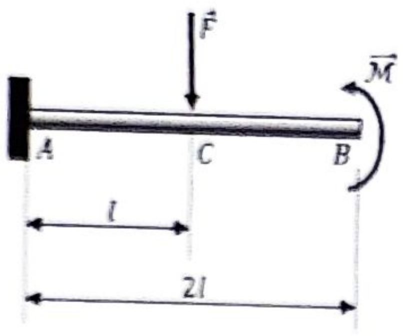
Ex. 2



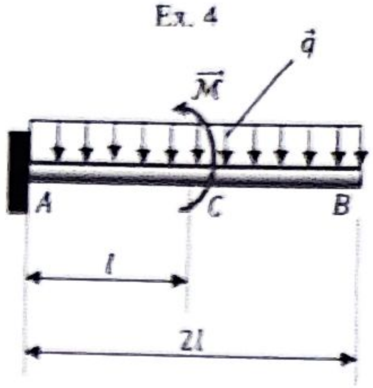
Ex. 3



Ex. 4



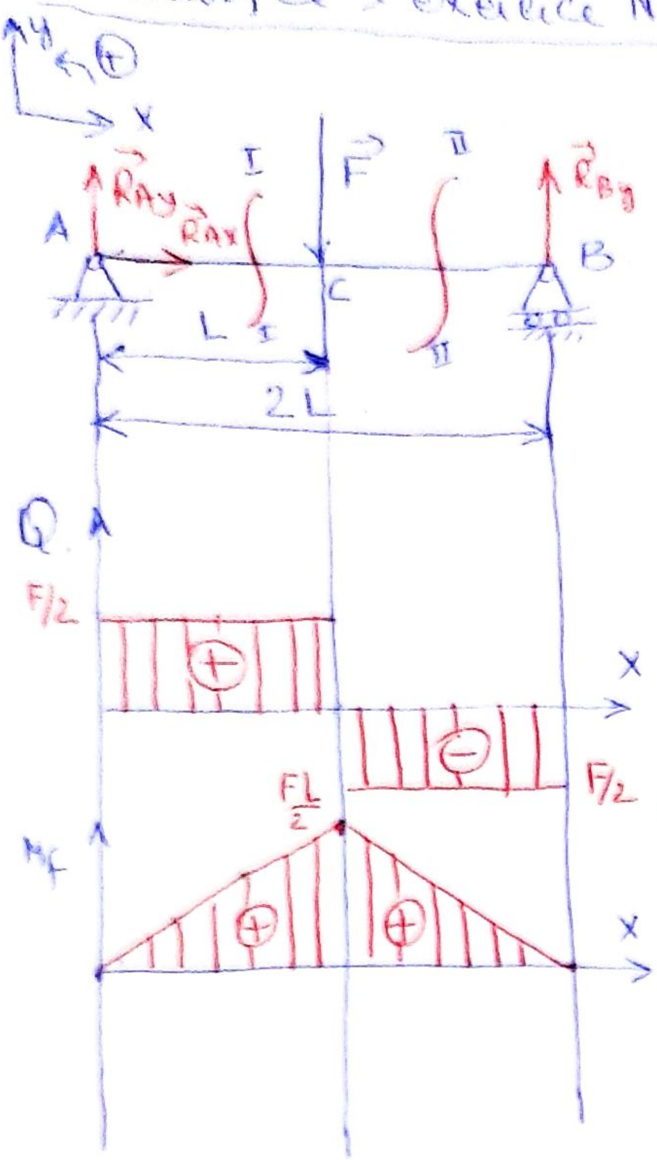
Ex. 5



Ex. 6

Chargés de cours — L. Kherredine — A. Chemami

Solution de l'exercice N° 1 du TD série 5.1 (Flexion)



Détermination des réactions $\vec{R}_{Ay}, \vec{R}_{Ax}, \vec{R}_{By}$

a) équations d'équilibres

$\sum F_{ext}/x = 0 \Rightarrow R_{Ax} = 0$. (1)

$\sum F_{ext}/y = 0 \Rightarrow R_{Ay} - F + R_{By} = 0$ (2)

$\sum M/A = 0 \Rightarrow -F.L + R_{By}.2L = 0$ (3)

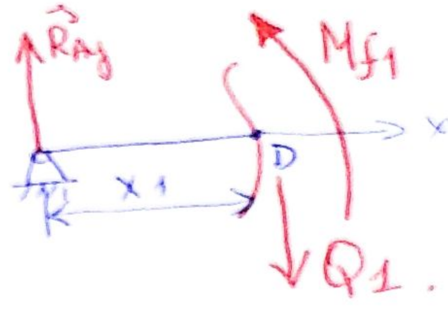
de l'équation (3) on tire :

$R_{By} = \frac{F.L}{2L} = \frac{F}{2}$

de l'équation (2) on tire R_{Ay} :

$R_{Ay} = F - R_{By} = \frac{2F}{2} - \frac{F}{2} = \frac{F}{2}$.

Tronçon I $\quad 0 \leq x_1 \leq L$



$\sum F/y = 0 \Rightarrow R_{Ay} - Q_1 = 0 \Rightarrow$

$Q_1 = R_{Ay}$.

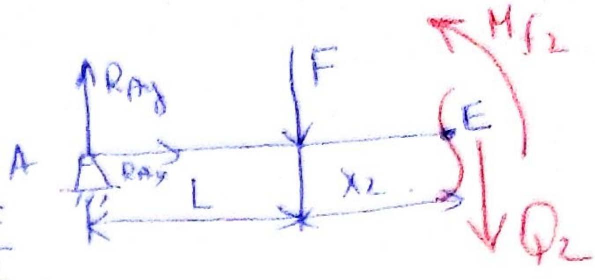
$\sum M/D = 0 \Rightarrow M_{f1} - R_{Ay}.x_1 = 0$

$M_{f1} = R_{Ay}.x_1$

pour $x_1 = 0 \Rightarrow M_{f1} = 0$

$x_1 = L \Rightarrow M_{f1} = R_{Ay}.L = \frac{FL}{2}$.

Tronçon II $0 \leq x_2 \leq L$



$\sum F_{ext/y} = 0 \Rightarrow R_{Ay} - F - Q_2 = 0$

$Q_2 = R_{Ay} - F = \frac{F}{2} - F = -\frac{F}{2}$

$\sum M/E = 0 \Rightarrow -R_{Ay}\cdot(L+x_2) + F\cdot x_2 + M_{f2} = 0$

$M_{f2} = R_{Ay}(L+x_2) - Fx_2$

pour $x_2 = 0$ $M_{f2} = R_{Ay}.L$

$x_2 = L$ $M_{f2} = 2L R_{Ay} - F.L = \cancel{2}L.\frac{F}{\cancel{2}} - F.L = 0$

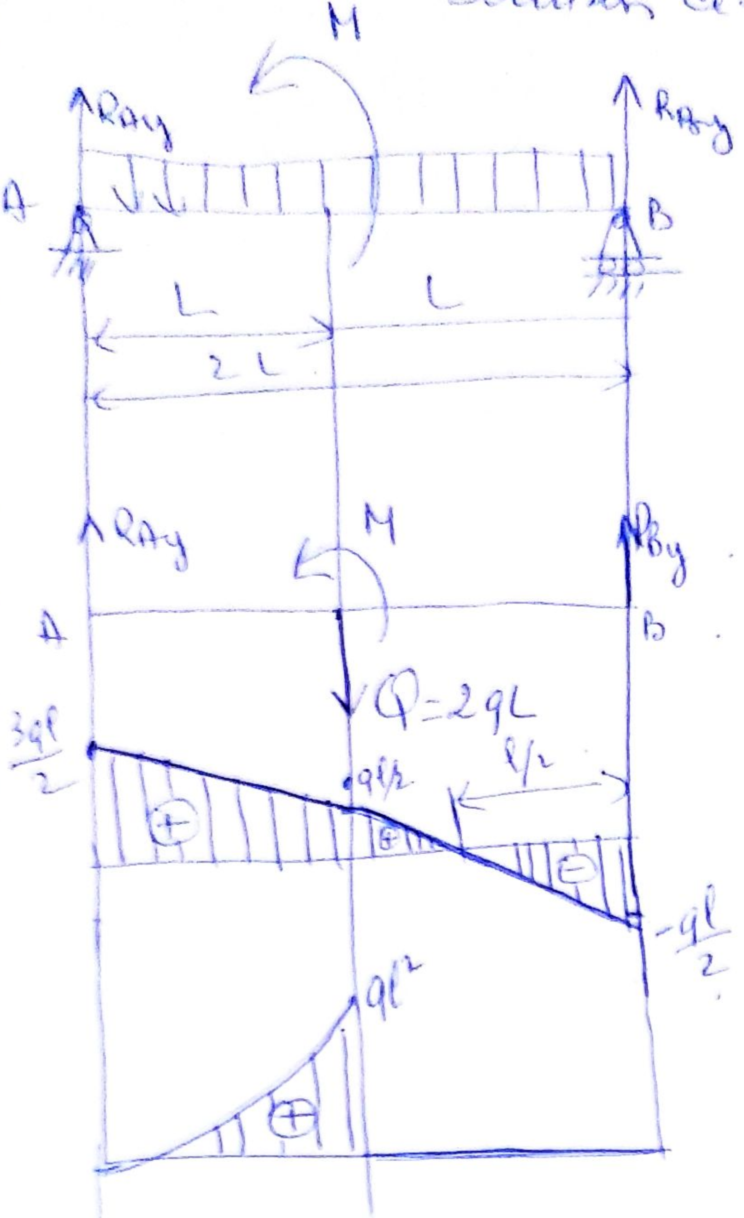
Determination de la position des sections dans lesquelles on a:

a) Flexion pure ($M_f \neq 0$ et $Q = 0$) pas de flexion pure

b) Cisaillement pure ($M_f = 0$ et $Q \neq 0$) $\Rightarrow$ Section située $x = 0$ et $x = 2L$.

Solution de l'exercice n° 3 de la série 5.1

Flexion



$\Sigma F_{ext}/y = 0 \Rightarrow$

$R_{Ay} - Q + R_{By} = 0$ (1)

$\Sigma M/A = 0 \Rightarrow$

$M - 2qL \cdot L + R_{By} \cdot 2L = 0$

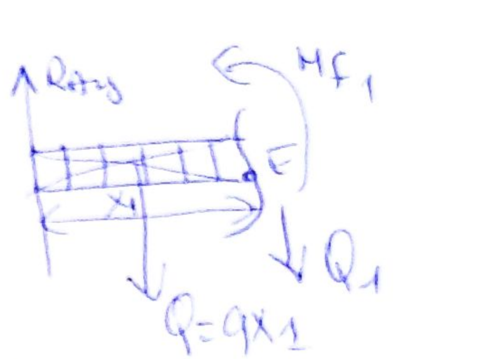
$$R_{By} = \frac{2ql^2 - M}{2L}$$

$$R_{By} = \frac{2ql^2 - ql^2}{2L} = \frac{ql^{\cancel{2}}}{2\cancel{L}}$$

$$R_{By} = \frac{ql}{2}.$$

$$R_{Ay} = Q - R_{By} = 2ql - \frac{ql}{2}.$$

$$R_{Ay} = \frac{3ql}{2}.$$



tronçon I $\quad 0 \leq x_1 \leq L$

$\Sigma F_{ext}/y = 0 \Rightarrow R_{Ay} - qx_1 - Q_1 = 0$

$Q_1 = -qx_1 + R_{Ay} = R_{Ay} - qx_1$

pour $x_1 = 0 \Rightarrow Q_1 = R_{Ay} = 3ql/2$.

$x_1 = L \Rightarrow Q_1 = \frac{3ql}{2} - \frac{2q \cdot L}{2} = \frac{ql}{2}$.

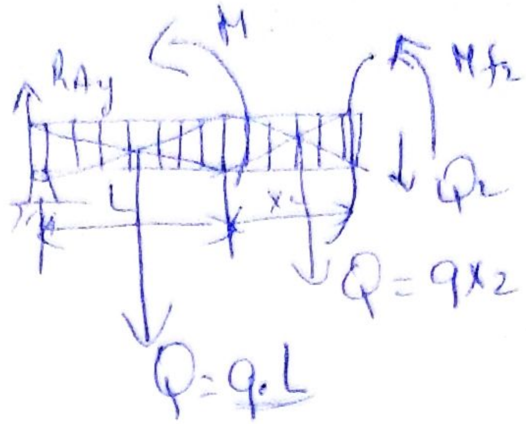
$\Sigma M/E = 0 \Rightarrow Mf_1 + q\frac{x_1^2}{2} - R_{Ay} \cdot x_1 = 0$

$Mf_1 = R_{Ay} \cdot x_1 - q\frac{x_1^2}{2}$

pour $x_1 = 0 \quad Mf_1 = 0$

$x_1 = L \quad Mf_1 = \frac{3ql^2}{2} - \frac{ql^2}{2} = \frac{\cancel{2}ql^2}{\cancel{2}} = ql^2$.

Trouçon II



$\sum F_{ext}/y = 0 \Rightarrow$

$R_{Ay} - qL - qx_2 - Q_2 = 0$

$Q_2 = R_{Ay} - qL - qx_2$

para $x_2 = 0 \Rightarrow \frac{3ql}{2} - \frac{2ql}{2} - 0 = \frac{ql}{2}$

$x_2 = L \Rightarrow \frac{3ql}{2} - qL - qL = \frac{3ql}{2} - \frac{4ql}{2} = -\frac{ql}{2}$

$\sum M/E = 0 \Rightarrow -R_{Ay}(L + x_2) + ql\left(\frac{l}{2} + x_2\right) + q\frac{x^2}{2} + M + M_{f_2} = 0$

$M_{f_2} = R_{Ay}(L + x_2) - qL\left(\frac{l}{2} + x_2\right) - q\frac{x^2}{2} - M$

para $x_2 = 0$ $M_{f_2} = R_{Ay}.L - q\frac{L^2}{2} - M$

$M_{f_2} = \frac{3ql}{2}.l - \frac{ql^2}{2} - ql^2 = \frac{2ql^2 - ql^2}{2} = ql^2 - ql^2 = 0$

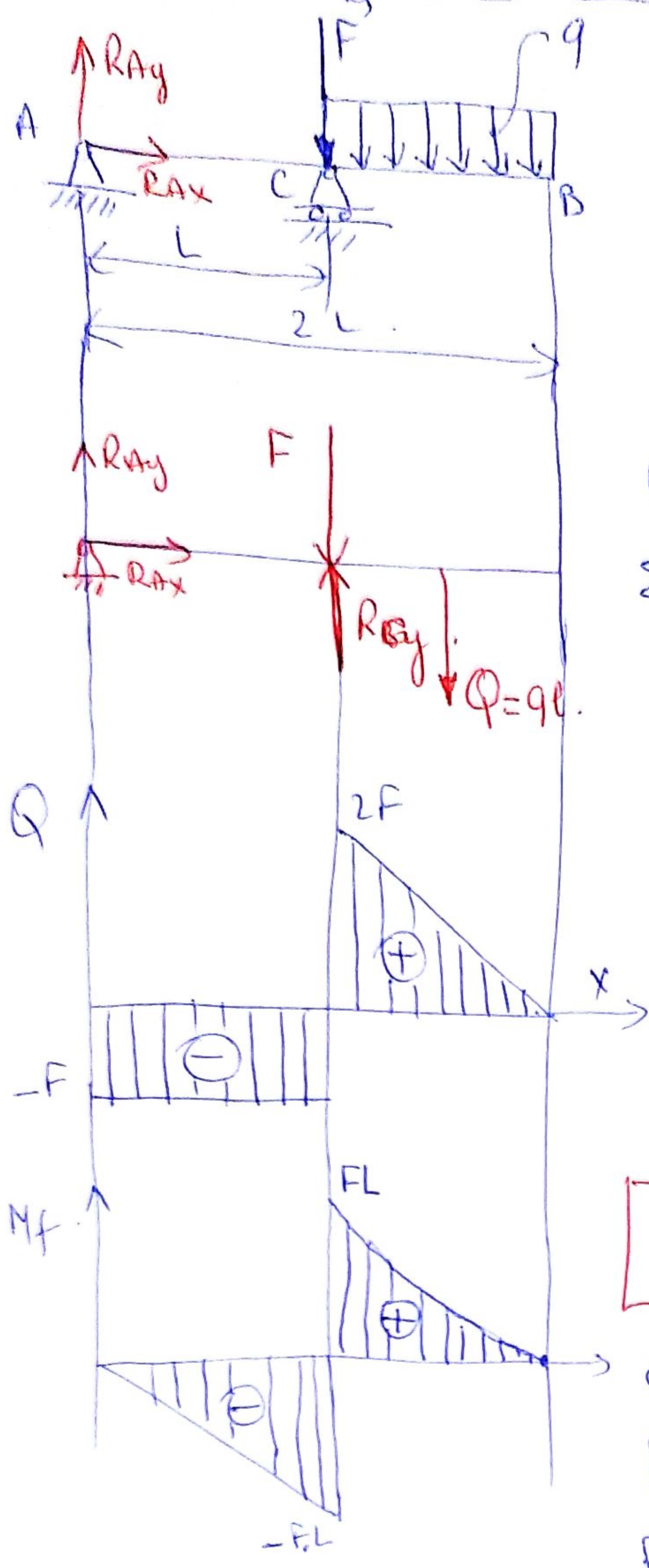
para $x_2 = L$. $M_{f_2} = \frac{3ql}{2}(L + L) - qL\left(\frac{L}{2} + L\right) - q.\frac{L^2}{2} - ql^2$

$M_{f_2} = \frac{3ql}{2}.2l - qL\left(\frac{3L}{2}\right) - \frac{ql^2}{2} - \frac{2ql^2}{2}$

$M_{f_2} = \frac{6ql^2}{2} - \frac{3ql^2}{2} - \frac{2ql^2}{2} - \frac{ql^2}{2} =$

0

Solution de l'exercice n° 4 de la serie 5.1. (Flexion



détermination des réactions aux appuis A et B.

Équations des équilibres.

$\sum F_{ext/y} = 0$

$R_{Ay} - F + R_{cy} - Q = 0$ (1)

$\sum M/_A = 0 \Rightarrow -F.L + R_{cy}.L - ql(l+\frac{l}{2}) = 0$

$-FL + R_{cy}L - q\frac{3l^2}{2} = 0$ (2)

$R_{cy} = \frac{FL + 3ql^2/2}{L}$

$R_{cy} = \frac{2FL + 3qL^2}{2L} =$

$R_{cy} = \frac{2FL + 3.\frac{2F}{\cancel{L}}.L^{\cancel{2}}}{2L}$

$\boxed{R_{cy}} = \frac{8F\cancel{L}}{\cancel{2}\cancel{L}} = \boxed{4F}$

de (1) on tire ;

$R_{Ay} = F - R_{cy} + Q$

$R_{Ay} = F - 4F + \frac{2F}{\cancel{L}}.\cancel{L}$

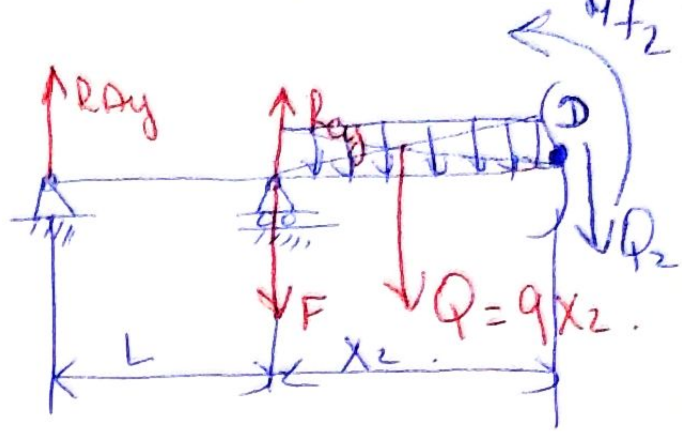
$\boxed{R_{Ay} = -F}$

Vérification :

$R_{Ay} - F + R_{cy} - Q = 0$.

$-F - F + 4F - 2F = 0$

Tronçon II ~~Méthode~~ $0 \le X_2 \le L$.



$\sum F_{ext}/y = 0 \Rightarrow$

$R_{Ay} + R_{cy} - F - Q - Q_2 = 0$.

$Q_2 = R_{Ay} + R_{cy} - F - Q$.

$Q_2 = -F + 4F - F - qX_2$

pour $\boxed{X_2 = 0} \Rightarrow Q_2 = \boxed{2F}$

pour $\boxed{X_2 = L} \Rightarrow Q_2 = -F + 4F - F - \frac{2F}{\cancel{L}}.\cancel{L} = \boxed{0}$

$\sum M/D = 0 \Rightarrow -R_{Ay}(L+X_2) - R_{cy}X_2 + FX_2 + q\frac{X^2}{2} + M_{f2} = 0$

$M_{f2} = R_{Ay}(L+X_2) + R_{cy}X_2 - FX_2 - q\frac{X^2}{2}$

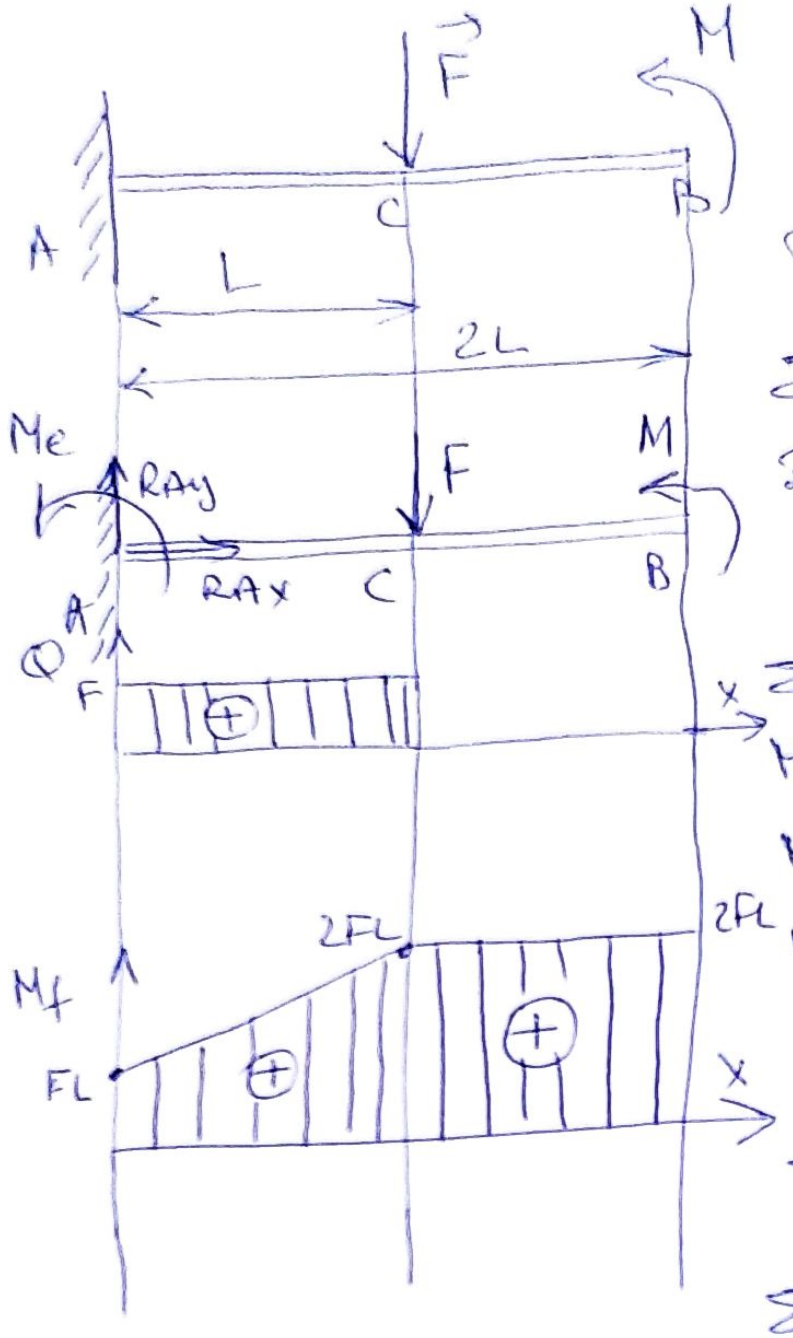
pour $\boxed{X_2 = 0}$ $M_{f2} = R_{Ay}.L = \boxed{FL}$.

$\boxed{X_2 = L}$ $M_{f2} = -F(L+L) + 4F.L - F.L - \frac{\cancel{2}F}{\cancel{L}}.\frac{L^{\cancel{2}}}{\cancel{2}}$

$M_{f2} = -2FL + 4FL - 2FL = \boxed{0}$.

pour avoir une flexion pure il faut avoir
$M_f \neq 0$ et $Q = 0$.
pour avoir un cisaillement pur il faut avoir
$M_f = 0$ et $Q \neq 0$. dans notre cas on
a ni l'un ni l'autre. 2/2.

Solution de l'exercice N° 5 de la serie 5.1 (flexion).



Détermination des Réactions au pt A.

$\sum F_{ext}/y = 0 \Rightarrow$

$R_{Ay} - F = 0 \Rightarrow$

$R_{Ay} = F.$

$\sum M/A = 0 \Rightarrow$

$M_e - FL + M = 0.$

$M_e = FL - M$

$M_e = FL - \frac{2F}{L} \cdot L^2 = -FL.$

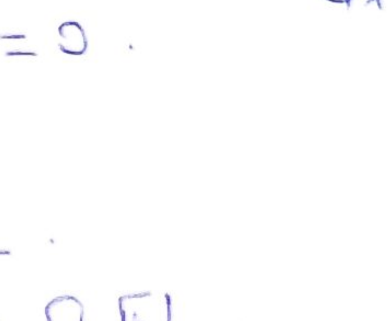
$M_e = -FL.$

<u>Tronçon I</u> $0 \leq x_1 \leq L$

$\sum F_{ext}/y = 0 \Rightarrow$

$R_{Ay} - Q_1 = 0$

$R_{Ay} = Q_1 = F$



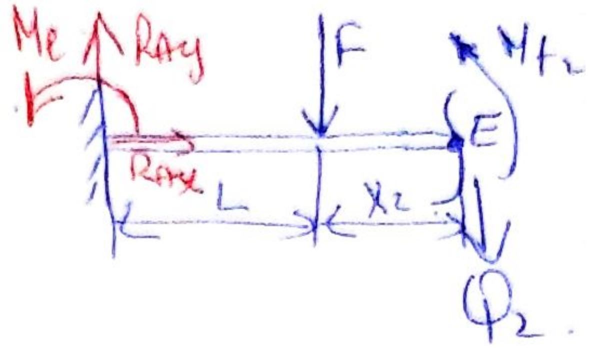
$\sum M/D = 0 \Rightarrow M_{f1} - R_{Ay} \cdot x_1 + M_e = 0.$

$M_{f1} = R_{Ay} \cdot x_1 - M_e.$

pour $x_1 = 0$ $\quad M_{f1} = -M_e = FL.$

$x_1 = L$ $\quad M_{f1} = F.L + FL = 2FL.$

Tronçon II $\quad 0 \leq X_2 \leq L$.



$\sum F_{ext}/y = 0 \Rightarrow R_{Ay} - F - \varphi_2 = 0$

$\varphi_2 = R_{Ay} - F$.

$\varphi_2 = F - F = 0$.

$\sum M/_E = 0 \Rightarrow -R_{Ay}(L + X_2) + F X_2 + M_{f2} + M_e = 0$

$M_{f2} = R_{Ay}(L + X_2) - F X_2 - M_e$

pour $X_2 = 0 \Rightarrow M_{f2} = R_{Ay}.L + FL = 2FL$.

$X_2 = L \Rightarrow M_{f2} = R_{Ay}(L + L) - \cancel{F.L} + \cancel{F.L}$

$M_{f2} = 2FL - \cancel{FL} + \cancel{FL}$.

$M_{f2} = 2FL$.

Cisaillement pur $(M_f = 0$ et $\varphi \neq 0)$ pas de cisaillement pur ~~pour X_1~~

flexion pur $(M_f \neq 0$ et $\varphi = 0)$ pour

X_1 compris entre L et $2L$.