$$S_3 = \begin{cases} x + 2y - z + t = 4 \\ x - y + z + 2t = 4 \\ 3x - y + 2z - t = 6 \\ x - y - z + 3t = 3 \end{cases}$$

$$\underbrace{\begin{pmatrix} 1 & 2 & -1 & 1 \\ 1 & -1 & 1 & 2 \\ 3 & -1 & 2 & -1 \\ 1 & -1 & -1 & 3 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}}_{\cancel{X}} = \underbrace{\begin{pmatrix} 4 \\ 4 \\ 6 \\ 3 \end{pmatrix}}_{b}$$

$$\det A = 1 \begin{vmatrix} -1 & 1 & 2 \\ -1 & 2 & -1 \\ -1 & -1 & 3 \end{vmatrix} - 2 \begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & -1 \\ 1 & -1 & 3 \end{vmatrix} - 1 \begin{vmatrix} 1 & -1 & 2 \\ 3 & -1 & -1 \\ 1 & -1 & 3 \end{vmatrix} - \begin{vmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{vmatrix} = \frac{37}{70}$$

donc ~~en~~ le système est de Cramer, On utilise la méthode de Cramer.

$$x = \frac{\begin{vmatrix} 4 & 2 & -1 & 1 \\ 4 & -1 & 1 & 2 \\ 6 & -1 & 2 & -1 \\ 3 & -1 & -1 & 3 \end{vmatrix}}{\det A} = \frac{74}{37} = 2$$

$$z = \frac{\begin{vmatrix} 1 & 2 & 4 & 1 \\ 1 & -1 & 4 & 2 \\ 3 & -1 & 6 & -1 \\ 1 & -1 & 3 & 3 \end{vmatrix}}{\det A} = \frac{37}{37}$$

$$y = \frac{\begin{vmatrix} 1 & 4 & -1 & 1 \\ 1 & 4 & 1 & 2 \\ 3 & 6 & 2 & -1 \\ 1 & 3 & -1 & 3 \end{vmatrix}}{\det A} = \frac{37}{37} = 1$$

$$t = \frac{\begin{vmatrix} 1 & 2 & -1 & 4 \\ 1 & -1 & 1 & 4 \\ 3 & -1 & 2 & 6 \\ 1 & -1 & -1 & 3 \end{vmatrix}}{\det A} = \frac{37}{37} = 1$$

$$(x, y, z, t) = (2, 1, 1, 1)$$