on cherche une solution particulière par la méthode des variation ; $y_p = \alpha(x)\, y_1 + \beta(x)\, y_2$

$$\begin{cases} \alpha' y_1 + \beta' y_2 = 0 \\ \alpha' y_1' + \beta' y_2' = \dfrac{g(x)}{a} \end{cases}$$

(1) $y'' - 3y' + 2y = (x+1)\, e^{2x}$

$y'' - 3y' + 2y = 0$

$r^2 - 3r + 2 = 0$ $\qquad \Delta = 3^2 - 4(1)(2) = 1 > 0$

$r_1 = \dfrac{3-1}{2} = 1 \qquad r_2 = \dfrac{3+1}{2} = 2$

$y_H = \alpha e^{x} + \beta e^{2x}$

la solution particulière : $y_p = \alpha(x) y_1 + \beta(x) y_2$

on pose $y_1 = e^{x} \Rightarrow y_1' = e^{x}$
$y_2 = e^{2x} \Rightarrow y_2' = 2e^{2x}$

$$\Leftrightarrow \begin{cases} \alpha' y_1 + \beta' y_2 = 0 \\ \alpha' y_1' + \beta y_2' = \dfrac{g(x)}{a} \end{cases}$$

$$\Rightarrow \begin{cases} \alpha' e^{x} + \beta' e^{2x} = 0 \quad \text{---} (1) \\ \alpha' e^{x} + 2\beta' e^{2x} = (x+1)\, e^{2x} \quad \text{---} (2) \end{cases}$$

$(2)-(1) \Leftrightarrow \beta' \cancel{e^{2x}} = (x+1)\, \cancel{e^{2x}} \Leftrightarrow \beta' = x+1 \Rightarrow \boxed{\beta(x) = \frac{x^2}{2} + x}$

β' dans (1) $\Leftrightarrow$ $\cancel{\alpha' y_1}$ $\alpha' e^{x} + (x+1)\, e^{2x} = 0$

$\alpha' = -(x+1)\, e^{x} \Rightarrow \dfrac{d\alpha}{dx} = -(x+1)\, e^{x}$

$\int d\alpha = -\int (x+1)\, e^{x}\, dx$