$$S_2 = \begin{cases} x + 2y + 5z = 5 \\ 2x + 3y - z = -2 \\ 5x + 8y + 3z = 1 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & -1 \\ 5 & 8 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} \Leftrightarrow Ax = b$$

$$\det(A) = \begin{vmatrix} \overset{+}{1} & \overset{-}{2} & \overset{+}{5} \\ \overset{-}{2} & \overset{+}{3} & \overset{-}{-1} \\ \overset{+}{5} & \overset{+}{8} & 3 \end{vmatrix} = \begin{vmatrix} 3 & -1 \\ 8 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & -1 \\ 5 & 3 \end{vmatrix} + 5\begin{vmatrix} 2 & 3 \\ 5 & 8 \end{vmatrix}$$

$$= (9+8) - 2(6+5) + 5(16-15) = 0$$

le système n'est pas de Cramer, la solution n'est pas unique.

$$\begin{array}{c} L_1 \\ L_2 \\ L_3 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 5 & 5 \\ 2 & 3 & -1 & -2 \\ 5 & 8 & 3 & 1 \end{array}\right) \xrightarrow[L'_3 = 5L_1 - L_3]{L'_2 = 2L_1 - L_2} \left(\begin{array}{ccc|c} 1 & 2 & 5 & 5 \\ 0 & 1 & 11 & 12 \\ 0 & 2 & 22 & 24 \end{array}\right)$$

$$\xrightarrow[L''_3 = 2L'_2 - L'_3]{} \left(\begin{array}{ccc|c} 1 & 2 & 5 & 5 \\ 0 & 1 & 11 & 12 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x + 2y + 5z = 5 \Rightarrow x = -2y - 5z + 5 \\ y + 11z = 12 \Rightarrow y = 12 - 11z \end{cases}$$

$$x = -2(12 - 11z) - 5z + 5 = -24 + 22z - 5z + 5$$

$$x = -19 + 17z \quad (x, y, z) = (-19 + 17z, 12 - 11z, z)$$

$$y = 12 - 11z \qquad z \text{ libre.}$$