Ex03:

① $\begin{cases} x - 2y + z = -3 \\ x - y + z = 1 \\ -2x + 2y - z = 2 \end{cases}$

$\Rightarrow \begin{pmatrix} 1 & -2 & 1 \\ 1 & -1 & 1 \\ -2 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \Leftrightarrow Ax = b$

$x = A^{-1} b$

$A^{-1} = \frac{1}{\det(A)} (\mathrm{cof}(A))^t$

$\det(A) = 1 \begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix} - \begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} + (-1) \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$

$= 0 - (2-4) - (-1+2) = 2 - 1 = 1 \neq 0$

donc la matrice A est inversible.

$\mathrm{cof}\,A = \begin{pmatrix} +\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix} \\ -\begin{vmatrix} -2 & 1 \\ 2 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} \\ +\begin{vmatrix} -2 & 1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix}$

$(\mathrm{cof}\,A)^t = \begin{pmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$ donc $A^{-1} = \frac{1}{1} \begin{pmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1\times -3 + 0\times 1 + 2\times -1 \\ -3\times -1 + 1\times 1 + 0\times 2 \\ 0\times -3 + 2\times 1 + 1\times 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix}$

$x = A^{-1} \times b$ donc $x = 1, y = 4, z = 4$