$$\frac{d\beta}{dx} = \frac{3}{2}\cos 4x + \frac{3}{2}\cos 2x$$

$$\int d\beta = \frac{3}{2}\int \cos 4x\,dx + \frac{3}{2}\int \cos 2x\,dx$$

$$\boxed{\beta = \frac{3}{8}\sin 4x + \frac{3}{4}\sin 2x + c}$$

$$\begin{cases} \alpha' \cos^2 x + \beta' \cos \sin x = 0 \\ \alpha' \sin^2 x - \beta' \cos \sin x = 3\cos 3x \sin x \end{cases} \quad (+)$$

$$\alpha'\left[\cos^2 + \sin^2\right] = 3\cos 3x \sin x$$

$$\alpha' = 3\left[\frac{1}{2}\left(\left[\sin(3+1)x - \sin(3-1)x\right]\right)\right]$$

$$\alpha' = \frac{3}{2}\sin 4x - \frac{3}{2}\sin 2x$$

$$d\alpha = \frac{3}{2}\int \sin 4x\,dx - \frac{3}{2}\int \sin 2x\,dx$$

$$\boxed{\alpha = -\frac{3}{8}\cos 4x + \frac{3}{4}\cos 2x + c}$$

$$y_p = \alpha \cos x + \beta \sin x$$

$$= \left[\frac{3}{4}\cos 2x - \frac{3}{8}\cos 4x\right]\cos x + \left[\frac{3}{8}\sin 4x + \frac{3}{4}\sin 2x\right]\sin x$$

$$= \frac{3}{4}\cos 2x \cos x - \frac{3}{8}\cos 4x \cos x + \frac{3}{8}\sin 4x \sin x$$

$$+ \frac{3}{4}\sin 2x \sin x$$

$$= \frac{3}{8}\left(\cancel{\cos 3x} + \cos x\right) - \frac{3}{16}\left(\cos 5x + \cos 3x\right) + \frac{3}{16}\left(\cos 3x - \cos 5x\right)$$

$$+ \frac{3}{8}\left(\cos x - \cancel{\cos 3x}\right) = \frac{3}{4}\cos x + \cancel{\frac{3}{8}\cos} - \frac{3}{8}\cos 5x$$

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