Exo 4!

(1) $y'' + y = 3\cos 3x$.

$y'' + y = 0 \Rightarrow r^2 + 1 = 0$; $r^2 = -1 = i^2$; $r = \mp i$

donc la solution : $y_H = \alpha \cos x + \beta \sin x$.

on cherche la sol particulire par la méthode des variation : $y_p = \alpha(x)\, y_1 + \beta(x)\, y_2$

$$\begin{cases} \alpha' y_1 + \beta' y_2 = 0 \\ \alpha'_1 y'_1 + \beta' y'_2 = \dfrac{g(x)}{a} \end{cases}$$

on pose $y_1 = \cos x$ $y_2 = \sin x$; $y'_1 = -\sin x$; $y'_2 = \cos x$

$$\begin{cases} \alpha' \cos x + \beta' \sin x = 0 \\ -\alpha' \sin x + \beta' \cos x = 3\cos 3x \end{cases}$$

$\cos a \sin b = \frac{1}{2}\left[\sin(a+b) - \sin(a-b)\right]$

$\cos a \cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$

$\sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$

$\int \cos ax\,dx = \frac{1}{a}\sin ax + c$.

$\int \sin ax\,dx = -\frac{1}{a}\cos ax + c$

$$\begin{cases} \alpha' \cos x \sin x + \beta' \sin^2 x = 0 \\ -\alpha' \cos x \sin x + \beta' \cos^2 x = 3\cos 3x \cdot \cos x \end{cases}$$

$\beta'(\cos^2 + \sin^2) = 3\cos 3x \cos x$

$\beta' = 3\left[\frac{1}{2}(\cos 4x + \cos 2x)\right]$