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Partial Differential Equations

Multiple symmetric solutions for a Neumann problem with lack of compactness

Solutions symétriques multiples pour un problème de Neumann sans compacité

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A R T I C L E   I N F O

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A B S T R A C T

The existence of multiple cylindrically symmetric solutions for a class of non-autonomous elliptic Neumann problems in a strip-like domain of the Euclidean space is investigated. The proof combines a recent compactness result and the Palais symmetric critically principle. A concrete application illustrates the main abstract result of this Note.

1. Introduction

Let $\mathcal{O} \subset \mathbb{R}^m$ be a bounded domain with smooth boundary and set $\Omega := \mathcal{O} \times \mathbb{R}^n$. Define the space of cylindrically symmetric functions by

$$W_c^{1,p}(\Omega) := \{u \in W^{1,p}(\Omega):\ u(x,\cdot) \text{ is radially symmetric for all } x \in \mathcal{O}\}.$$

The aim of this Note is to establish the existence of multiple cylindrically symmetric weak solutions for the following non-autonomous elliptic Neumann problem:

$$\begin{cases} -\Delta_p u + |u|^{p-2}u = \lambda\alpha(x,y)f(u) & \text{in } \Omega \\ \partial u/\partial\nu = 0 & \text{on } \partial\Omega. \end{cases} \tag{P}$$

Here ν denotes the outward unit normal to $\partial\Omega$, $p > m+n$ is a real number, and $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$. We assume that $f : \mathbb{R} \to \mathbb{R}$ is a continuous function, α is a nonnegative cylindrically symmetric function, and λ is a positive real parameter.

In the present Note, just requiring a suitable oscillating behaviour of the potential $F(\xi) := \int_0^\xi f(t)\,dt$, we are able to find a precise interval of values of the parameter λ for which problem (P) admits at least three cylindrically symmetric weak solutions.

Assume $\alpha \in L^1(\Omega)$ is a nonnegative cylindrically symmetric function such that for some $\tau > 0$.

$$\operatorname{essinf}_{(x,y)\in\mathcal{O}\times B_n(0,\tau/2)} \alpha(x,y) > 0, \tag{1}$$

where $B_n(0,\tau/2)$ denotes the open ball in $\mathbb{R}^n$ centred in zero and radius $\tau/2$.

We say that $u \in W^{1,p}(\Omega)$ is a weak solution of problem (P) if for all $v \in W^{1,p}(\Omega)$,

$$\int_\Omega |\nabla u(x,y)|^{p-2}\nabla u(x,y)\cdot\nabla v(x,y)\,dx\,dy + \int_\Omega |u(x,y)|^{p-2}u(x,y)v(x,y)\,dx\,dy = \lambda\int_\Omega \alpha(x,y)f(u(x,y))v(x,y)\,dx\,dy.$$

- aim : but

- able : capable
- Assume : Supposer
- Admit : admettre
- At least : au moins
- Weak : faible
- radius : rayon
- Stands for : être
- fulfilled : satisfaite

- strip : bande
- investigate : étudier
- Combine : Combiner
- Main : principal
- Smooth boundary : frontière régulière, lisse
- outward : vers l'extérieur
- Just : seulement
- behaviour : comportement