④ $I_4 = \int \frac{dx}{1+\sin x + \cos x}$

On applique la règle de Bioche.

On pose $\begin{cases} t = \tan\frac{x}{2} & dx = \frac{2}{1+t^2}\,dt. \\ \sin x = \frac{2t}{1+t^2} & \\ \cos x = \frac{1-t^2}{1+t^2} & \tan x = \frac{2t}{1-t^2}. \end{cases}$

donc: $I_4 = \int \frac{\frac{2}{1+t^2}\,dt}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} = \int \frac{\frac{2}{1+t^2}\,dt}{\frac{1+t^2+2t+1-t^2}{1+t^2}}$

$= \int \frac{2\,dt}{2+2t} = \int \frac{dt}{1+t} = \ln|1+t| + C.$

$$\boxed{I_4 = \ln\left|1+\tan\left(\frac{x}{2}\right)\right| + C}$$

$I_5 = \int \frac{1}{1+a^2\sin x}\,dx$. on pose $t = \tan x$.

$dt = \frac{dx}{\cos^2 x}$.

$I_5 = \int \frac{1/\cos^2 x}{\frac{1}{\cos^2 x}+a^2\tan^2 x}\,dx = \int \frac{dt}{\frac{1}{\cos^2 x}+a^2t^2}$

On a: $\frac{1}{\cos^2 x} = 1+\tan^2 x = 1+t^2$.

donc.

$I_5 = \int \frac{dt}{1+t^2+a^2t^2} = \int \frac{dt}{1+(1+a^2)t^2} = \int \frac{dt}{1+\left(\sqrt{1+a^2}\,t\right)^2}$