Exo 5:

On a: $\int \sin^p x \cos^q x \, dx$.

1. si p impair on pose $u = \cos x$.

2. si q impair on pose $u = \sin x$.

3. Si p et q impair on fait un changement $u = \cos x$ ou $u = \sin x$.

4. si p et q pair on pose $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$.

(1) - $I_1 = \int \sin^{2019} x \cos^3 x \, dx$.

$p = 2019$ et $q = 3$ sont impairs.

donc on pose $u = \sin x$. $du = \cos x dx \Rightarrow dx = \frac{du}{\cos x}$

$$I_1 = \int u^{2019} \cdot \cos^3 x \frac{du}{\cos x} = \int u^{2019} \cos^2 x \, du.$$

On a. $\cos^2 x = 1 - \sin^2 x$ car $\cos^2 x + \sin^2 x = 1$

$$I_1 = \int u^{2019}(1 - u^2) \, du = \int u^{2019} du - \int u^{2021} du$$

$u^n \rightarrow \frac{u^{n+1}}{n+1}$

$$I_1 = \frac{u^{2020}}{2020} - \frac{u^{2022}}{2022} + C.$$

$$\boxed{I_1 = \frac{\sin^{2020} x}{2020} - \frac{\sin^{2022} x}{2022} + C}$$