Solution serie n° 1

Ex1

$[\rho] = \frac{[m]}{[V]} = \frac{M}{L^3} = ML^{-3}$

$[F] = [m][a] = MLT^{-2}$

$[q] = [i][t] = I \cdot T$

$E_c = \frac{1}{2} m v^2$

$[E_c] = [m][v^2] = ML^2T^{-2}$

$u = R \cdot i \qquad [R] = \frac{[u]}{[i]}.$

$W = q \cdot u. \qquad [u] = \frac{[W]}{[q]}.$

$[R] = \frac{[W]}{[i][q]} = \frac{ML^2T^{-2}}{I \cdot I T}$

$[R] = ML^2T^{-3}I^{-2}$

Ex2

$[R] = \frac{[P] \cdot [V]}{[T]}$

$[P] = \frac{[F]}{[S]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}.$

$[R] = \frac{ML^{-1}T^{2}L^3}{\theta} = ML^{2}T^{2}\theta^{-1}$

$[R] = ML^2T^2\theta^{-1}$

Ex3

$[F] = [\eta]^x [r]^y [v]^z.$

$MLT^{-2} = (ML^{-1}T^{-1})^x L^y (LT^{-1})^z$

$MLT^{-2} = M^x L^{y+z-x} T^{-x-z}$

$x = 1$

$-x-z = -2 \implies z = 2 - x = 1$

$y+z-x = 1 \qquad y = 1 + x - z$

$y = 1 + 1 - 1 = 1$

$\implies F = k\,\eta\, r\, v.$

Ex6

$u = x^2 + y^2 + z^2$

$v = x^2 + xy^2 + \sin y.$

$\frac{\partial u}{\partial x} = 2x \quad \frac{\partial u}{\partial y} = 2y \quad \frac{\partial u}{\partial z} = 2z$

$\frac{\partial v}{\partial x} = 2x + y^2$

$\frac{\partial v}{\partial y} = 2xy + \cos y$

$f = \frac{f_1 f_2}{f_1 - f_2 - \ell}.$

$dF = \frac{\partial F}{\partial f_1} \cdot df_1 + \frac{\partial F}{\partial f_2} df_2$

$\frac{\partial F}{\partial f_1} = \frac{f_2(f_1 - f_2 - \ell) + f_1 f_2}{(f_1 - f_2 - \ell)^2}$

$\frac{\partial F}{\partial f_2} = \frac{f_1(f_1 - f_2 - \ell) + f_1 f_2}{(f_1 - f_2 - \ell)^2}$

$dF = \frac{-f_2(f_2 + \ell)}{(f_1 - f_2 - \ell)^2} df_1 + \frac{f_1(f_1 - \ell)}{(f_1 - f_2 - \ell)^2} df_2$

Ex5

$F = x^2 \cos y$

$dF = \frac{\partial F}{\partial x} \cdot dx + \frac{\partial F}{\partial y} \cdot dy$

$\frac{\partial F}{\partial x} = 2x \cos y$

$\frac{\partial F}{\partial y} = -x^2 \sin y$

$dF = 2x \cos y \cdot dx - x^2 \sin y \cdot dy$

$\Delta F = 2x|\cos y|\,\Delta x + x^2|\sin y|\,\Delta y$

$\frac{dF}{F} = \frac{2x \cos y}{x^2 \cdot \cos y} \cdot dx - \frac{x^2 \sin y}{x^2 \cos y} dy.$

$\frac{\Delta F}{F} = 2\frac{\Delta x}{x} + |\mathrm{tg}\, y|\,\Delta y$

$\log F = 2 \log x + \log(\cos y)$

$\frac{dF}{F} = 2\frac{dx}{x} - \frac{\sin y}{\cos y} dy$

$\frac{\Delta F}{F} = 2\frac{\Delta x}{x} + |\mathrm{tg}\,\alpha|\,\Delta y$

Ex4

$$[T^2] = \frac{[m]}{[k]}$$

$$[k] = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L} = MT^{-2}.$$

$$[T^2] = \frac{\cancel{M}}{\cancel{M}T^{-2}} = T^2 \cdot \text{ Juste.}$$

$$* \; T^2 = 4\pi^2 \frac{k}{m}$$

$$[T^2] = \frac{[k]}{[m]} = \frac{\cancel{M}T^{-2}}{\cancel{M}} = T^{-2} \text{ fausse}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad T^2 = \frac{4\pi^2 m}{k}$$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\times(3{,}14)^2 \times 210.10^{-3}}{(1{,}1)^2} = 6{,}84 \; (N/m)$$

$$\log k = \log 4\pi^2 + \log m - 2\log T$$

$$\frac{dk}{k} = \frac{dm}{m} - 2\frac{dT}{T}$$

$$\frac{\Delta k}{k} = \frac{\Delta m}{m} + 2\frac{\Delta T}{T}$$

$$\frac{\Delta k}{k} = \frac{5}{210} + \frac{2\times 0{,}1}{1{,}1} = 0{,}024 + 0{,}182$$

$$\frac{\Delta k}{k} = 0{,}21 \qquad \frac{\Delta k}{k} = 21\,\%.$$

$$\Delta k = k \times 0{,}21 = 6{,}84 \times 0{,}21 = 1{,}44 \; N/m$$

$$k = (6{,}84 \mp 1{,}44) \; N/m$$

Ex7

$$Z = (R^2 + 4\pi^2 f^2 L^2)^{1/2}$$

$$\log Z = \frac{1}{2}\log(R^2 + 4\pi^2 f^2 L^2)$$

$$\frac{dZ}{Z} = \frac{1}{2(R^2 + 4\pi^2 L^2 f^2)}\left[2R\,dR + 8\pi^2 f^2 L\, d\ldots + 8\pi^2 L^2 f^2 d\ldots\right]$$

$$\frac{\Delta Z}{Z} = \frac{1}{2(R^2 + 4\pi^2 L^2 f^2)}\left[2R\,\Delta R + 8\pi^2 f^2 L\,\Delta\ldots + 8\pi^2 L^2 f^2 \Delta f\ldots\right]$$

$$\frac{\Delta Z}{Z} = 0{,}28 \qquad \frac{\Delta Z}{Z} = 28\,\%$$

$$\Delta Z = Z \times 0{,}28 = 6{,}92 \times 0{,}28 = 1{,}93\ldots$$

$$Z = (6{,}92 \mp 1{,}93) \; \Omega \, .$$