Convergence of Improper Integrals

The p-test for improper integrals: The improper integral

$$\int_1^\infty \frac{1}{x^p}\,dx$$

converges to $\frac{1}{p-1}$ if $p > 1$ and diverges if $p \leq 1$.

The Comparison Theorem: Suppose that f and g are continuous functions with $f(x) \geq g(x) \geq 0$ for $x \geq a$.

(a) If $\int_a^\infty f(x)\,dx$ is convergent, then $\int_a^\infty g(x)\,dx$ is convergent.

(b) If $\int_a^\infty g(x)\,dx$ is divergent, then $\int_a^\infty \,dx$ is divergent.

A similar result holds for improper integrals with discontinuous (unbounded) integrands.

The Absolute Comparison Theorem: Suppose that $\int_a^\infty |f(x)|\,dx$ converges. Then $\int_a^\infty f(x)\,dx$ also converges.

Examples.

1. Does
$$I = \int_1^\infty \frac{1}{x^5+1}\,dx$$
converge or diverge?

2. Does
$$I = \int_1^\infty \frac{2x+1}{\sqrt{x}-\frac{1}{2}}\,dx$$
converge or diverge?

3. Does
$$I = \int_1^\infty \frac{1+e^{-x}}{x}\,dx$$
converge or diverge?

4. Does
$$I = \int_1^\infty \frac{\cos^2 x}{1+x^2}\,dx$$
converge or diverge?

5. Does
$$I = \int_1^\infty \frac{\sin x}{x^2}\,dx$$
converge or diverge?