Determining surface temperature for an axisymmetric inverse heat conduction problem

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Introduction

The inverse heat conduction problems (IHCB) arises from many physical and engineering problems. It is often impossible to directly measure the desired physical quantity. In this situation ,one way to measure this quantity is to solve a boundary value inverse heat conduction problem.

We willl consider an axisymmetric inverse problem for the heat aquation inside the cylinder $a \leq r \leq b$

Direct problem

We assume that $u(a,t) = f(t)$ is known and we consider the following direct problem:

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial u^2}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}, \quad r \in (a,b), \quad t \geq 0, \\ u(a,r) = f(t), \\ \frac{\partial u}{\partial r}(b,t) = 0, \quad t \geq 0 \\ u(r,0) = 0 \quad r \in [a,b]. \end{cases} \tag{1}$$

construction of the solution

We use the Laplace transform for the representation of the solution: Let

$$F = \mathcal{L}(f) \Leftrightarrow F(s) := \int_0^{+\infty} e^{-st} f(t) dt \tag{2}$$

The inverse Laplace transform is given by the complex inversion formula

$$f(t) = \mathcal{L}^{-1}(F)(t) = \frac{1}{2\pi i} \int_{\sigma-\infty}^{\sigma+\infty} e^{st} F(s) ds, \quad t > 0 \tag{3}$$

Theorem

- *Assume that* $f(t) \in C^1$*,such that* $f(0) = 0$ *for* $t \geq T$*. Then the series* (12) *converge in* $L^2(]a,b)[)$ *for all*$t \geq 0$ *and defines one solution of the problem* (1) *in* H.

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